

Basic Trigonometric Identities

Basic Identities

$$\sin \theta = \frac{y}{r} = \frac{\text{opp}}{\text{hyp}} \quad \cos \theta = \frac{x}{r} = \frac{\text{adj}}{\text{hyp}} \quad \tan \theta = \frac{y}{x} = \frac{\text{opp}}{\text{adj}}$$

$$\csc \theta = \frac{1}{\sin \theta} \quad \sec \theta = \frac{1}{\cos \theta} \quad \cot \theta = \frac{1}{\tan \theta}$$

Memorize these!



Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

Memorize at least the first one

the other two can be derived by dividing the first equation by $\sin^2 \theta$ or $\cos^2 \theta$

Memorize at least sum and difference of sine and cosine

If you forget the tangent identities - $\tan(a+b)$ can be determined by dividing $\sin(a+b)$ by $\cos(a+b)$. Likewise, for $\tan(a-b)$.

Sum and Difference Identities

$$\sin(a + b) = \sin a \cos b + \cos a \sin b$$

$$\sin(a - b) = \sin a \cos b - \cos a \sin b$$

$$\cos(a + b) = \cos a \cos b - \sin a \sin b$$

$$\cos(a - b) = \cos a \cos b + \sin a \sin b$$

$$\tan(a + b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

$$\tan(a - b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

Double-Angle Identities

$$\sin 2\theta = 2\sin \theta \cos \theta$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\cos 2\theta = 2\cos^2 \theta - 1$$

$$\cos 2\theta = 1 - 2\sin^2 \theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

Memorize at least sine and cosine double angle identities

If you forget the tangent identity - $\tan 2\theta$ can be determined by dividing $\sin 2\theta$ by $\cos 2\theta$.

Know at least the sine, cosine, and tangent identities

Sine and tangent are odd; cosine is even.

Even and Odd Identities

$$\sin(-x) = -\sin x \quad \csc(-x) = -\csc x$$

$$\cos(-x) = \cos x \quad \sec(-x) = \sec x$$

$$\tan(-x) = -\tan x \quad \cot(-x) = -\cot x$$

Basic Trigonometric Identities

Power Reducing Formulas

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

$$\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$$

$$\tan^2 \theta = \frac{1 - \cos 2\theta}{1 + \cos 2\theta}$$

Memorize at least sine and cosine power reducing identities

If you forget the tangent identity – $\tan^2\theta$ can be determined by dividing $\sin^2\theta$ by $\cos^2\theta$.

Memorize the sine, cosine, and at least one of the tangent half angle identities

Half Angle Identities

$$\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}}$$

$$\cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{2}}$$

$$\tan \frac{\theta}{2} = \frac{\sin \theta}{1 + \cos \theta} = \frac{1 - \cos \theta}{\sin \theta} = \pm \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}}$$

Co-function Identities

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta \quad \tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta \quad \csc\left(\frac{\pi}{2} - \theta\right) = \sec \theta$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta \quad \cot\left(\frac{\pi}{2} - \theta\right) = \tan \theta \quad \sec\left(\frac{\pi}{2} - \theta\right) = \csc \theta$$

Know at least the sine and cosine versions

Product to Sum Formulas

$$\sin a \sin b = \frac{1}{2} [\cos(a - b) - \cos(a + b)]$$

$$\cos a \cos b = \frac{1}{2} [\cos(a - b) + \cos(a + b)]$$

$$\sin a \cos b = \frac{1}{2} [\sin(a - b) + \sin(a + b)]$$

Know how to apply Product to Sum and Sum to Product Formulas

Sum to Product Formulas

$$\sin a + \sin b = 2 \sin\left(\frac{a + b}{2}\right) \cos\left(\frac{a - b}{2}\right)$$

$$\sin a - \sin b = 2 \cos\left(\frac{a + b}{2}\right) \sin\left(\frac{a - b}{2}\right)$$

$$\cos a + \cos b = 2 \cos\left(\frac{a + b}{2}\right) \cos\left(\frac{a - b}{2}\right)$$

$$\cos a - \cos b = -2 \sin\left(\frac{a + b}{2}\right) \sin\left(\frac{a - b}{2}\right)$$