

Basic Trigonometric Identities

Basic Identities

$$\sin \theta = \frac{y}{r} = \frac{opp}{hyn}$$

$$\cos\theta = \frac{x}{r} = \frac{adj}{hyp}$$

$$\sin \theta = \frac{y}{r} = \frac{opp}{hyp}$$
 $\cos \theta = \frac{x}{r} = \frac{adj}{hyp}$ $\tan \theta = \frac{y}{x} = \frac{opp}{adj}$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

Memorize these!



Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$
$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

Memorize at least the first one

the other two can be derived by dividing the first equation by $sin^2\theta$ or $cos^2\theta$

Memorize at least sum and difference of sine and cosine

If you forget the tangent identities - tan(a+b) can be determined by dividing sin(a+b) by cos(a+b). Likewise, for tan(a-b).

Sum and Difference Identities

$$\sin(a+b) = \sin a \cos b + \cos a \sin b$$

$$\sin(a - b) = \sin a \cos b - \cos a \sin b$$

$$cos(a + b) = cos a cos b - sin a sin b$$

$$\cos(a - b) = \cos a \cos b + \sin a \sin b$$

$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

Double-Angle Identities

$$\sin 2\theta = 2\sin \theta \cos \theta$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\cos 2\theta = 2\cos^2 \theta - 1$$

$$\cos 2\theta = 1 - 2\sin^2\theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

Memorize at least sine and cosine double angle identities

If you forget the tangent identity – $tan2\theta$ can be determined by dividing $\sin 2\theta$ by $\cos 2\theta$.

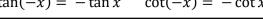
Know at least the sine, cosine, and tangent identities

Sine and tangent are odd; cosine is even.

Even and Odd Identities

$$\sin(-x) = -\sin x$$
 $\csc(-x) = -\csc x$
 $\cos(-x) = \cos x$ $\sec(-x) = \sec x$

$$\tan(-x) = -\tan x \qquad \cot(-x) = -\cot x$$







Basic Trigonometric Identities

Power Reducing Formulas

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$
$$\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$$
$$\tan^2 \theta = \frac{1 - \cos 2\theta}{1 + \cos 2\theta}$$

Memorize at least sine and cosine power reducing identities

If you forget the tangent identity – $tan^2\theta$ can be determined by dividing $sin^2\theta$ by $cos^2\theta$.

Memorize the sine, cosine, and at least one of the tangent half angle identities

Half Angle Identities $\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}}$ $\cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{2}}$ $\tan \frac{\theta}{2} = \frac{\sin \theta}{1 + \cos \theta} = \frac{1 - \cos \theta}{\sin \theta} = \pm \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}}$

Co-function Identities

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta \quad \tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta \quad \csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta$$
$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta \quad \cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta \quad \sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$$

Know at least the sine and cosine versions

Product to Sum Formulas

$$\sin a \sin b = \frac{1}{2} [\cos(a-b) - \cos(a+b)]$$

$$\cos a \cos b = \frac{1}{2} [\cos(a-b) + \cos(a+b)]$$

$$\sin a \cos b = \frac{1}{2} [\sin(a-b) + \sin(a+b)]$$

Know how to apply Product to Sum and Sum to Product Formulas

Sum to Product Formulas $\sin a + \sin b = 2\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$ $\sin a - \sin b = 2\cos\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$ $\cos a + \cos b = 2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$ $\cos a - \cos b = -2\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$

