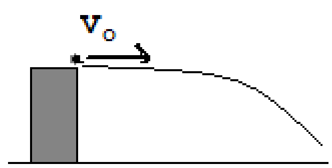
Physical Science
Projectile Motion

When a cannon ball is shot from the top of a building or a baseball is hit off of the end of a bat, the moving object goes in a curved path which must be described in two directions, the horizontal (x) direction and the vertical (y) direction. Such examples of two dimensional kinematics are commonly referred to as **projectile motion**, where the moving projectile is thrown with an initial velocity in the horizontal direction. As the object travels through the air, each direction of motion is determined by separate conditions. As you work projectile motion problems, you should keep in mind which direction to use when obtaining the requested information, and therefore which set of conditions will determine how the projectile moves.

One particularly straightforward example of projectile motion is the case of a horizontally projected object. In this case, the initial velocity is only in the horizontal (x) direction. This simplifies the problem because there is no initial velocity in the y-direction. For example, if you kick a ball horizontally at a speed from the top of a 70 m building, the situation can be sketched as follows:



Note that to describe the position of the ball at any instant in time, you have to find not only how far the ball has fallen (the y-component) as well as how far the ball travels away from the building. These two parts of the motion are best solved for separately as we work projectile problems. In other words, the x- and y-components of the position are independent of each other, and can be worked as individual rectilinear kinematics problems. You simply have to know how the motion of the object changes for each component in order to solve for positions, times and velocities.

As an example, consider the following problem:

EXAMPLE: A ball is thrown at 15 m/s horizontally from the top of 30 meter tall tower. Calculate a) the time it takes for the ball to land and hit the ground, and b) how far from the base of the building the ball lands (a distance called the **range** of the projectile).

For this type of problem, it is important to ask yourself which direction the question deals with before attempting to solve it. In the solution shown below, be sure to think about how the time connects the two directions of motion.

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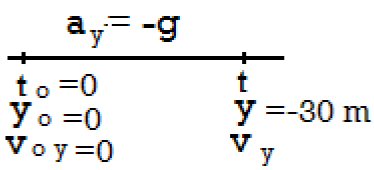
a) time of fall for the projectile t_{fall} :

The time it takes for the projectile to fall to the ground deals with the y-direction of motion (how long it takes for the ball to travel from the top to the bottom of the building). A line diagram showing the kinematical parameters which describe the y-position of the ball would look like:



Notice that the initial velocity in this direction is zero since you are throwing the ball in the x-direction (not up or down at all). In addition, since the ball has fallen downward, below the starting point, it is a negative position. The formula which describes this situation can be taken from rectilinear kinematics: $x = v_o t + 1/2\ a\ t^2$. To match the line diagram, we can change the formula for the y-direction:

$$y = v_{oy}t + 1/2\ a_y\ t^2 = -\ 1/2 g\ t^2 \quad \text{since } v_{oy} = 0$$

substituting in the values from the line diagram, we get the relation

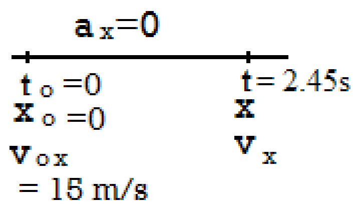
$$-30\text{m} = -1/2\ (10\ \text{m/s2})\ t^2 \text{ which can be simplified to } t^2 = 6\ s^2$$

Taking the square root of both sides of the equation, we get the time of fall $t_{fall} = 2.45$ s

The time calculated above is valid whether the ball is projected with an initial horizontal velocity, or whether it is just dropped from rest!

b) the range of the projectile:

The **range** of the projectile represents the distance the ball travels away from the building. For this problem, you want to find how far the ball travels in the x-direction, in the amount of time it takes to hit the ground. The line diagram which describes the position in the x-direction looks like:



In this case, there is no acceleration since no forces act on the ball (it is falling in the absence of air friction). Also, since the ball is thrown in the x-direction only, the entire initial speed is written in this direction. This describes how the ball moves in the x-direction as it fall to the ground.

The kinematic equation which describes the position of the ball for this situation is: $x = v_{ox}t + 1/2\ a_x\ t^2$ which can be simplified using the fact that the acceleration in this direction is zero:

$$x = v_{ox}t = (15\ \text{m/s})(2.45\ \text{s}) = 36.8\ \text{m}$$

which is the range of the ball.