

Physics

Momentum in Collisions

Momentum is a physical quantity which represents the difficulty in stopping an object. As a mass moves at a faster and faster speed, it gains more energy and must be met with a larger force to bring it to rest. Not surprisingly, the momentum can be calculated by multiplying the mass of the object times its velocity ($p = m v$). Therefore a car moving at 70 mph will have less momentum than a truck moving at the same speed, and will cause less damage if brought to a sudden stop in a collision.

Impulse-momentum theorem

In order to change the momentum of an object, a force must be applied causing it to accelerate from the initial speed v_o according to Newton's second law of motion ($F_{\text{NET}} = ma$). The change in momentum resulting from an external force is called the impulse ($I = mv - mv_o$) and is related to the force applied through the **Impulse-Momentum Theorem**:

$$I = F t_c = mv - mv_o .$$

Here F is the external force applied and t_c is the amount of time the force is in contact with the object. This formula applies to a variety of situations where an external force is used to change the velocity of an object such as the force of a racket which changes the speed and direction of a ball or a soccer ball accelerated by a player's foot. In general very little is known about the mass or velocity of the object applying the external force.

Consider the following example:

Ex. #1. A pitched baseball of mass 0.2 kg arrives at home plate with a speed of 40 m/s and is batted straight **backwards** to the pitcher with a return speed of 60 m/s.

- a) What is the change in the ball's momentum?
- b) If the ball is in contact with the bat for 0.5 seconds, with what average force did the bat hit the ball?

Solution for a) The change in momentum is the same as the impulse delivered to ball in this example $I = mv - mv_o$. Plugging in we get for the impulse $I = (0.2 \text{ kg})(-60 \text{ m/s}) - (0.2 \text{ kg})(40 \text{ m/s}) = -12 \text{ kg m/s} - 8 \text{ kg m/s} = -20 \text{ kg m/s}$. Note the velocity of the ball is in the negative direction since it is opposite to the direction of the ball's original motion. So the change in momentum is simply the impulse which is $I = -20 \text{ kg m/s}$, which is in the direction opposite to the original velocity.

Solution for b) The impulse-momentum theorem describes the impulse (or change in momentum) in terms of the force applied during impact. Using the middle part of the formula given above, we can write a formula for the force as $I = F t_c \rightarrow -20 \text{ kg m/s} = F (0.5 \text{ s})$ where t_c has been replaced with the time the ball is in contact with the bat. Dividing both sides by 0.5 s yields the final answer for the force of contact between the ball and bat, $F = -40 \text{ N}$. Again the force applied is negative since it accelerated the ball to go in the negative direction.

Physics

Conservation of momentum

When two objects collide together, there are internal forces that act to change their velocity after they are in contact with each other. Since the resultant forces are action-reaction pairs governed by Newton's third law, they act in opposite directions so the two objects accelerate in the direction opposite to their original motion. As a consequence of the interaction one of the particles will generally gain momentum, while the other loses some of the momentum it originally had. In the absence of other external forces like friction or gravity, the exchange of momentum causes the total momentum of the two particle system to be exactly the same as it was before the collision; in other words, the momentum lost by one object equals the momentum gained by the other. The **law of conservation of momentum** states this fact as follows: the total momentum of a closed system is the same before and after a collision in which no external forces are present ($P_{\text{tot}}^{\text{before}} = P_{\text{tot}}^{\text{after}}$). Here the total momentum is just the sum of the momentum for each object $P_{\text{tot}} = m_1v_1 + m_2v_2$. The conservation of momentum holds for situations ranging from billiard ball collisions to automobiles crashing together. Even though external forces exist in these cases, they are usually insignificant compared to the large internal forces from the impact.

Problems which involve the conservation of momentum generally have two objects moving toward each other which collide and move away or stick together and move together. Note that the masses and initial velocities of each object must be known so that the total momentum can be calculated. By equating the total momentum before a collision to the final momentum, information can be gained about the final velocities of each. The case where the two particles stick together is known as a **perfectly (or totally) inelastic collision**. The problem becomes simplified because both of the masses m_1 and m_2 have the same speed after they stick together.

The following example of a perfectly inelastic collision examines the final speed of a lump of clay which overtakes a second, and the two stick together after they make contact.

Ex. #2. A 100 g lump of clay moving eastward at 10 m/s collides with a 150 g lump of clay moving in the same direction at 5 m/s. The two coalesce(stick together) after impact. With what velocity does the combined mass move after impact?

Solution: First of all, remember the SI unit for mass is kilograms, so the masses can be rewritten as $m_1 = 0.1$ kg, and $m_2 = 0.15$ kg. The masses and velocities of each are given in the problem, so the total momentum before the collision can be calculated as

$$P_{\text{tot}}^{\text{before}} = m_1v_1 + m_2v_2 = (0.1 \text{ kg})(10 \text{ m/s}) + (0.15 \text{ kg})(5 \text{ m/s}) = 1.75 \text{ kg m/s}.$$

Notice that the two lumps of clay are moving in the same direction, so there is no change in sign for the velocities (they both move in the positive direction). Also, the only forces given in the problem are from the collision, so they represent internal action-reaction forces. Since this is the case, the total momentum will be conserved, or will be the same before and after the collision, so we can write the total momentum after the collision as

$$P_{\text{tot}}^{\text{after}} = m_1v_1 + m_2v_2 = (0.1 \text{ kg})(v_f) + (0.15 \text{ kg})(v_f) = (0.35 \text{ kg})(v_f)$$

where we have used the fact that each part of the combined mass will have the same final velocity after they collide. Setting the momenta equal to each other, we can solve for the final velocity as

$$P_{\text{tot}}^{\text{before}} = P_{\text{tot}}^{\text{after}}$$

$$1.75 \text{ kg m/s} = (0.35 \text{ kg})(v_f)$$

and solving this we get $v_f = 5$ m/s. Note that the velocity of one of the masses increases, while the other slows down, showing how momentum is exchanged to produce the final velocity.