ACADEMIC SUPPORT CENTER
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Student's t-distribution

(Finding critical values for "t")

Student's *t*-distribution (or simply the ***t*-distribution**) is a probability distribution that arises in the problem of estimating the mean of a normally distributed population when the population standard deviation is unknown and has to be estimated from the data.

Table A-3 T Distribution

	∝					
Degrees of Freedom	.005 (one tail) .01 (two tail)	.01 (one tail) .02 (two tail)	.025 (one tail) .05 (two tail)	.05 (one tail) .10 (two tail)	.10 (one tail) .20 (two tail)	.25 (one tail) .50 (two tail)
1	63.657	31.821	12.706	6.314	3.078	1.000
2	9.925	6.965	4.303	2.920	1.886	.816
3	5.841	4.541	3.182	2.353	1.638	.765
4	4.604	3.747	2.776	2.132	1.533	.741
5	4.032	3.365	2.571	2.015	1.476	.727
6	3.707	3.143	2.447	1.943	1.440	.718
7	3.500	2.998	2.365	1.895	1.415	.711
8	3.355	2.896	2.306	1.860	1.397	.706
9	3.250	2.821	2.262	1.833	1.383	.703
10	3.169	2.764	2.228	1.812	1.372	.700

The number at the beginning of each row in the table above is defined as the degrees of freedom = $n - 1$. The decimal along the top is alpha = ∝, the level of significance. The numbers in the main body of the table are the critical values. Below are three examples of finding the critical values for t using the chart.

1) Find the critical value for a 95% confidence level where n = 10 and σ is unknown.
   Degrees of freedom = n – 1 = 9
   (alpha) ∝ = 1 - .95 = .05 in two tails (column # 4)
   T = 2.262 is the critical value for t.

2) Find the critical values for a 98% confidence level where n = 5 and σ is unknown.
   Degrees of freedom = n – 1 = 4
   (alpha) ∝ = 1 - .98 = .02 in two tails (column # 3)
   T = 3.747 is the critical value for t.

3) Find the critical values for a 90% confidence level where n = 9 and σ is unknown.
   Degrees of freedom = n – 1 = 8
   (alpha) ∝ = 1 - .90 = .10 in two tails (column # 5)
   T = 1.860 is the critical value for t.

Construct a Confidence Interval using the t-distribution

To use the t-distribution the standard deviation (σ) of the population is not known and the population data is approximately normally distributed. Below are two examples of constructing a confidence interval for a mean.

1) **Estimating cost to repair a car**: In crash tests of 10 cars, collision repair cost is found to have a distribution that is roughly bell-shaped, with a mean of $1786 and a standard deviation of $937. Construct a 99% confidence interval for the mean repair cost in all such vehicle collisions.

Define the variables:

n = 10	Number in the sample
s = 937	Standard deviation of the sample
$\bar{X}$ = 1786	Mean of the sample
n – 1 = 9	Degrees of freedom
α = 1 - .99=.01	Level of significance for a Confidence Interval of 99%

Use column #2 where α = .01 in two tails
Move down column # 1 to the degrees of freedom = 9 and read the number in the column #2
The critical value is t = 3.250 for a confidence level of 99%
Construct a Confidence Interval for the Estimate of μ where σ (sigma) is not known

$\bar{X} - E < \mu < \bar{X} + E$ Find the margin of error (E) $E = t_{\alpha/2}\dfrac{s}{\sqrt{n}} = 3.250\dfrac{937}{\sqrt{10}} = 962.99 \approx 963$

$1786 - 963 < \mu < 1786 + 963$

$823 < \mu < 2749$

2) **Estimating Car Pollution:** In a sample of 7 cars each car was tested for emissions of nitrogen-oxide and found the average emission was 0.121 grams per mile and the standard deviation is 0 .04 grams per mile. Construct a 98% confidence interval estimate of the mean amount of nitrogen-oxide emissions for all cars.

Define the variables:

n = 7	Number in the sample
s =0 .04	Standard Deviation of the sample
$\bar{X}$ =0 .121	Mean of the sample
n – 1 = 6	Degrees of freedom
α = 1 - .98=.02	Level of significance for a Confidence Interval 98%

Use column # 3 where α = .02 in two tails.
Move down column #1 to the degrees of freedom = 6 and read the number in the column #3.
The critical value is t = 3.143 for a confidence level of 98%
Construct a Confidence Interval for the Estimate of μ where σ (sigma) is not known

$\bar{X} - E < \mu < \bar{X} + E$ Find the margin of error (E) $E = t_{\alpha/2}\dfrac{s}{\sqrt{n}} = 3.143\dfrac{0.04}{\sqrt{7}} \approx 0.047$

$0.121 - 0.047 < \mu < 0.121 + 0.047$

$0.074 < \mu < 0.168$