

Constructing a Confidence Interval for a Normal Distribution

Finding Critical Numbers for Confidence Intervals

Confidence intervals can be used to estimate the proportion of a population who have a particular attribute

Table A-2 **NEGATIVE z Scores**

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
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-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
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To find the critical value for a confidence interval in a Normal Distribution, $z_{\alpha/2}$, first find $\alpha/2$. To do this convert the confidence level to a decimal and subtract it from 1, then divide by two. Because $\alpha/2$ is less than 0.5, you will find the z-score on the Negative side of the Standard Normal (z) Distribution, part of which is represented at the top of the page. Find the z-score that corresponds to $\alpha/2$ or to the nearest possible value. Note that even though the z-score is found on the Negative side of the Standard Normal (z) Distribution, that the critical number for a confidence interval is always positive. Alternatively, you can look up $1 - \alpha/2$ on the Positive side of the Standard Normal (z) Distribution. This will give you the exact same critical number.

Example: Find $z_{\alpha/2}$ for a 96% confidence interval. First, convert 96% to a decimal, subtract from 1 and divide by 2: $1 - .96 = .04$ divided by 2 yields .02. We want to find a z score corresponding to .0200. Looking at the proportions on the Negative z Scores table on the row labeled -2.0 we see .0202 in the column corresponding to .05 and .0197 in the column corresponding to .06. Since .0202 is closer to the desired value, we will use $-(2.0 + .05) = -2.05$. The critical number is positive because this is a confidence interval, so $z_{\alpha/2} = 2.05$.

Constructing a Confidence Interval for a population mean when σ is known:

The Standard Normal Distribution is used to estimate a population mean when two conditions are met. First, the sample size must be large, $n > 30$, or the population distribution must be normal. Second, there must be an

acceptable estimate for the standard deviation (σ) of the population. The following is an example of constructing a confidence interval for a mean.

Estimate the mean IQ score. A professional association of psychologists decides that it wants to estimate the mean IQ scores for its members using a 98% confidence level. A sample of 36 members takes the IQ test. The mean score is 109.4. IQ tests have a population standard deviation of $\sigma = 15.0$. This is a large sample where $n > 30$, and σ is known. These are the conditions needed to use the Standard Normal Distribution to find the critical values.

Define the variables:

Number in the sample

$$n = 36$$

Population standard deviation

$$\sigma = 15.0$$

Mean of the sample

$$\bar{X} = 109.4$$

Confidence Interval 98% means that $\alpha = 1 - .98 = .02$, so $\alpha/2 = .01$.

The critical number corresponding to a proportion to .0100 is approximately -2.33 .

Taking $z_{\alpha/2}$ to be positive yields

$$z_{\alpha/2} = 2.33$$

$$\bar{X} - E < \mu < \bar{X} + E \text{ where } E = z_{\alpha/2} \frac{\sigma}{\sqrt{n}} = 2.33 \frac{15.0}{\sqrt{36}} \cong 5.8$$

$$109.4 - 5.8 < \mu < 109.4 + 5.8$$

$$103.6 < \mu < 115.2$$

Constructing a Confidence Interval for a population proportion:

Samples are used to estimate the proportion of a population that posses one particular criteria. A sample is taken and the number of objects which meet the criteria are recorded. When $np > 5$ and $nq > 5$ where n is the size of the sample, p is the proportion of the population which meet the criteria and q is the proportion of the population which do not meet the criteria, the result is a Standard Normal Distribution. By dividing the resulting distribution by the number in the sample, the result is a Standard Normal Distribution that reflects the proportion of the samples which meet the criteria. Thus the Standard Normal Distribution is used to create confidence intervals for population proportions. The following is an example of constructing a confidence interval for a population proportion.

Estimate the proportion of pet owners. A pet store wants to estimate the proportion of households in a particular community who own pets with a confidence level of 97%. A sample of 150 households shows that 90 of them own pets.

Define the variables:

Number in the sample

$$n = 150$$

Proportion of the sample who own a pet

$$\hat{p} = \frac{90}{150} = 0.60$$

Proportion of the sample who do not own a pet

$$\hat{q} = 1 - 0.60 = 0.40$$

Confidence Interval 97% means that $\alpha = 1 - .97 = .03$, so $\alpha/2 = .015$.

The critical number corresponding to a proportion to .0150 is approximately -2.17 .

Taking $z_{\alpha/2}$ to be positive yields

$$z_{\alpha/2} = 2.17$$

$$\hat{p} - E < p < \hat{p} + E \text{ where } E = z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}} = 2.17 \sqrt{\frac{(0.6)(0.4)}{150}} \cong 0.087$$

$$0.600 - 0.087 < p < 0.600 + 0.087$$

$$0.513 < \mu < 0.687$$