## Statistics Tutorial, Formula Applications and TI-34 Instructions



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## Chapter 1

## Statistical Literacy and Critical Thinking

## Parameter and Statistic "What is a parameter, and what is a statistic?

A parameter is a numerical measurement describing some characteristic of a population. ( $p$ with $p$ )
A statistic is a numerical measurement describing some characteristics of a sample. (s with s)

## How do quantitative data and categorical data differ?

Quantitative data consists of numbers representing counts or measurement.
Categorical data can be separated into different categories' that are distinguished by some characteristic that is not numerical.

## How do discrete and continuous data differ?

The numbers of people surveyed in each of the next several Gallup polls
Discrete because it is quantitative and the answer will be countable
The exact heights of individuals in a sample of several statistics students
Continuous because there can be numerous results. Heights can be anywhere from 1 to 8 feet
The number of Super Bowl football games that must be played before one of the teams scores exactly 75 points

Discrete because it is quantitative.

## Identifying the Population

In a Gallup poll of 1010 adults in the United States, $55 \%$ of the respondents said that they used local TV stations daily as a source of news. Is the 1010 value a statistic or a parameter? Is the $55 \%$ value a statistic or a parameter? Describe the population.

The values of 1010 and $55 \%$ are both statistics because they are based on a sample.
The population would consist of all adults in the United States.

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## Chapter 2 and 3:

## EDIT Feature using the Calculator

EDIT - allows the entry (in columns) and modifications of data to be used with the CALC features.

## Press Data

Press Data a second time, and you may delete data that is already in the list columns. Highlight $L$ to be cleared and hit ENTER.

## Editing lists

If you want to remove an entry, move the up arrow to the data to be removed and press the delete button

## Changing existing entry

Change the 18 to 20. Press Data
Move the down arrow to the 20. Type18, and hit Enter.

The 18 will replace the 20 .

## Descriptive Statistics for one set of Raw Data

Find the midrange, mean, median, and mode for the given sample data.
Listed below are the ages of motorcyclists when they were fatally injured in 18 traffic crashes.


## Using the Calculator

Press Data If data exists in any columns
Press Data again

Clear

1: Clear L1
2. Clear L2

Clear the home screen by high lighting each $L$ that has data and hitting the ENTER button

Press Data again Enter all numbers into L1. Press ENTER after each entry until all the numbers are inputted.
$(10,12,13,20,18)$

Once all data is entered. Press $2^{\text {nd }}$, STAT,.

## 1: 1-Var Stats

## Press ENTER.

L1 should be high-lighted on the List. If another List is to be used use the right arrow to move to that list. FreqList should be high-lighted one

Use the down arrow to scroll down to Calc


STATS

1. 1 Var Stats

1-War Stat.s
List: L1
FreqList: one Calc:

Press ENTER for the results.

ACADEMIC SUPPORT C ENTER

## Summary Statistics for one set of Raw Data using Frequency Distribution

## Statistical Literacy and Critical Thinking

A set of data has a maximum value of 74 , a median value of 61 , and a minimum value of 48 .
Use the range rule of thumb to estimate the standard deviation, rounding to the nearest tenth.

$$
s=\frac{\text { range }}{4}=\frac{74-61}{4}=\frac{13}{4}=3.25=3.3
$$

## Analysis of Frequency Distribution

Identify, the class width, class midpoints and class boundaries.

| Age <br> (years) | Frequency | Mid <br> points | Class <br> boundaries |
| :--- | ---: | :--- | ---: |
| $9-11$ | $\mathbf{3}$ | $\mathbf{1 0}$ | $\mathbf{8 . 5 - 1 1 . 5}$ |
| $12-14$ | 2 | $\mathbf{1 3}$ | $\mathbf{1 1 . 5 - 1 4 . 5}$ |
| $15-17$ | 5 | $\mathbf{1 6}$ | $\mathbf{1 4 . 5 - 1 7 . 5}$ |
| $18-20$ | $\mathbf{3}$ | $\mathbf{1 9}$ | $\mathbf{1 7 . 5 - 2 0 . 5}$ |
| $21-23$ | $\mathbf{6}$ | 22 | $20.5-\mathbf{2 3 . 5}$ |
| $24-27$ | $\mathbf{4}$ | $\mathbf{2 5}$ | $\mathbf{2 3 . 5 - 2 6 . 5}$ |

## Class width = 3 (12-9)

The difference between the first numbers of two consecutive class intervals.
Class midpoints $=\frac{11+9}{2}=10$
The midpoint of a class is the sum of the first and last number in the class divided by 2.

## Class boundaries

The sum of the last number and first number of two consecutive class intervals divided by 2 . $\frac{11+12}{2}=11.5$

Is it possible to identify the actual age of the 5 people in age from 15-17 years?
No! The upper and lower limits include the specific ages. You know 5 people have ages between between 15-17 years, but you do not know their actual age.

## Using the Calculator

Given 6 class midpoints and there related frequencies, determine the mean, median, and mode.

## Press Data

Press ENTER,

Clear the home screen (see EDIT instructions page 4)
Enter the class midpoints of your data into L1 (10, 13, 16, 19, 22, 25).

Enter the frequencies of your class midpoints into L2 (3, 2, 5, 3, 6, 4).
Once all data is entered. Press $2^{\text {nd }}$, STAT

## 1: 1-Var Stats



EDIT CALC TESTS

1. 1 Var Stats

1-Var Stats

L1 appears in List.
FreqList should L2
Use the down arrow to scroll down to Calc

## List: L1 <br> FreqList:L2 <br> Calc

Press ENTER for results

Press the down arrow to see the remainder of the output.

1-Vars Stats
1: $n=23$
2: $\bar{x}=18.47826087$
3: $S x=5.007898899$
4: $\sigma x=4.897821740$
5: $\sum x=425$
6: $\Sigma x^{2}=8405$

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## Chapter 4:

## Basic Concepts of Probability

## Criteria for a Probability

Which of the following values may NOT be a probability?
a) $\frac{1}{3}$
b) -1
c. 0.87
d) 1.01
e) $75 \%$
$b$ and d Probabilities cannot be minus or $>1$

## Relative Frequency Probability

If a person is randomly selected, find the probability that his or her birthday is October 17. Ignore leap years.

$$
P(A)=\frac{\text { number of times } A \text { occurred }}{\text { number of times trial was repeated }}=1 / 365
$$

If a person is randomly selected, find the probability that his or her birthday is in October (has 31 days)

$$
P(A)=\frac{\text { number of times } A \text { occurred }}{\text { number of times trial was repeated }}=31 / 365
$$

## Multiplication Rule

A set of digits $\{0,1,2,3,4,5,6,7\}$ will be used to create a five-digit code. How many five-digit codes can be created if...
a. there are no restrictions? $\mathbf{8}^{5}=\mathbf{3 2 7 6 8}$
b. the digits may not be repeated? $\mathbf{8} \mathbf{X} \mathbf{7} \times \mathbf{6} \times 5 \times \mathbf{4}=\mathbf{6 7 2 0}$
c. the code must begin with a 3 and the digits may be repeated? $\mathbf{8}^{4}=4096$

Two different cards are drawn from a standard deck of 52 cards. What is the probability that the first card is a king and the second card is a queen?

$$
\frac{4 \text { kings }}{52} \times \frac{4 \text { queens }}{51}=\frac{16}{2652}=.00603
$$

A box of marbles has 3 yellow marbles, 2 green marbles, and 5 red marbles. Suppose that two marbles are selected without replacement. What is the probability that neither selected marble is green?

$$
\frac{8}{10} \times \frac{7}{9}=\frac{56}{90}=.6222
$$



## Permutations nPr

If a president and vice president are to be selected from 5 committee members how many permutations can be selected. In this example order is important so we use Permutations ${ }_{n} P_{r}$
${ }_{5} P_{2}=\frac{5!}{(5-2)!}=\frac{5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1}=\frac{120}{6}=20$

## Using the calculator

First enter "n"(5), then select PRB-then Scroll down to $\boldsymbol{n P r}$ and hit ENTER

Press 2 and hit ENTER again and the answer 20 appears
${ }_{5} P_{2}$

## Combinations ${ }_{n} C_{r}$

If 3 committee members are select from 5 employees how many
combinations can be selected. In this example order is not important so we use Combinations $={ }_{n} C_{r}$

Prb:
${ }_{n} C_{r}$
${ }_{5} C_{3}=\frac{5!}{(5-3)!(3!}=\frac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 1 \times 3 \times 2 \times 1}=\frac{120}{12}=10$
Using the calculator
First enter "n"(5), then select PRB-then Scroll down to ${ }_{n} C_{r}$ and hit ENTER

Press 3 and hit ENTER again and the answer 10 appears
${ }_{5} C_{3}$
ANS 10

## Statistical Thinking

During a certain week the number of late and on-time flights were counted for three airlines.

|  | Number of <br> On-time Flights | Number of <br> Late Flights | Total |
| :--- | :--- | :--- | :--- |
| Southern Airlines | 34 | 11 | 45 |
| Statewide Airlines | 28 | 3 | 31 |
| Westward Airlines | 30 | 8 | 38 |
| Total | 92 | 22 | 114 |

What is the probability that the flight was a Westward Airlines flight and was late?

$$
\frac{8}{114}=.0702
$$

What is the probability that the flight was late, given that the flight was on Westward Airlines?

$$
\frac{8}{38}=.2105
$$

An experiment has a sample space given by $S=\{1,2,3,4,5,6,7,8,9,10\}$. The events $D, E$, and $F$ are defined as $D=\{1,2,3,4\}, E=\{2,4,6,8,10\}$, and $F=\{3,5,7\}$.

Assume each outcome is equally likely:
Are events $D$ and $E$ disjoint (mutually exclusive)? No
Are events $D$ and $F$ disjoint (mutually exclusive)? No
Are events E and F disjoint (mutually exclusive)? Yes
Determine whether events G and H are independent or dependent.
G: Joe forgets to set his alarm clock. H: Joe is late for work the next day. Dependent
G: Milk is on sale. $\quad \mathrm{H}$ : Notebook paper is on sale. Independent
G: A student earns an "A" in English 1.
H: A student in English 1 is a girl. Independent

## Calculating Odds

A box contains 6 white chips, 3 blue chips, 2 yellow chips, and 1 green chip. A player randomly selects one chip from the box. If the player selects a blue chip, he/she wins $\$ 3$. If the player selects any other color chip, he/she loses $\$ 1$

Find the players odds of winning?

$$
\begin{aligned}
\text { odds } & =\frac{\# \text { favorable }}{\# \text { unfavorable }}=\frac{3}{9}=\frac{1}{3} \quad 1: 3 \\
u & =\sum x \bullet P(x)=3\left(\frac{3}{12}\right)+-1\left(\frac{9}{12}\right)=0
\end{aligned}
$$

When you give a casino $\$ 5$ for a bet on the "pass the line" in the game of craps, there is a 244/495 probability that you will win $\$ 5$ and a 251/495 probability that you will lose $\$ 5$.

What are your odds of winning?

$$
\text { odds }=\frac{\# \text { favorable }}{\# \text { unfavorable }}=244: 251
$$

What is your expected value?

$$
u=\sum x \bullet P(x)=5\left(\frac{244}{295}\right)+-5\left(\frac{251}{495}\right)=-.0 .0707 \text { cents }
$$

In the long run, how much do you lose for each dollar bet? $\frac{-0.0707}{5}=-0.014=1.4$ eents

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## Chapter 5:

## Parameters for Binomial Distribution

Among 945 births, there were 879 girls born to parents. Assuming boys and girls are equally likely, find the mean and standard deviation for the number of girls born in this group of 945 babies.
$u=n \bullet p \quad$ mean binomial
$\sigma=\sqrt{n \bullet p \bullet q}$ standard deviation binomial
$\sigma^{2}=n \bullet p \bullet q$ variance binomial

For this example, $n=945$ and $p=0.50 q=0.50$
Mean: $\quad u=n \bullet p \quad u=945 \bullet 0.50=472.5$ girls

Standard Deviation: $\quad \sigma=\sqrt{945 \bullet .5 \bullet .5}=15.3704=15.4$ girls
variance binomial $\sigma^{2}=n \bullet p \bullet q=945 \bullet 0.50 \bullet 0.50=236.25$
Find the minimum usual number and maximum usual number of girls born in this group of 945.

Minimum usual value: $\mu-2 \sigma=472.5-2(15.4)=441.7$ girls
Maximum usual value: $\mu+2 \sigma=472.5+2(15.4)=503.3$ girls

## Binomial Probability Distribution Values

Find the probability that 2 successes will occur out of 5 trials if the probability of success is 0.3.

$$
\begin{aligned}
& P(x)=\frac{n!}{(n-x)!x!} \bullet p^{x} q^{n-x} \text { binomial probability } \\
& Q=1.0-p=1.0-.3=.7 \\
& P(2)=\frac{5!}{(5-2)!2!} \bullet .3^{2} \cdot 7^{3}=.3087
\end{aligned}
$$

Find the probability that 2 or less successes will occur out of 5 trials if the probability of success is 0.3 . That is, find $P(x \leq 2)$. We need to find the probability of 2,1 , and 0 and add the totals.

$$
\begin{aligned}
& P(x)=\frac{n!}{(n-x)!x!} \cdot p^{x} q^{n-x} \text { binomial probability } \\
& Q=1.0-p=1.0-.3=.7 \\
& P(2)=\frac{5!}{(5-2)!2!} \times .3^{2} \times .7^{3}=.3087 \\
& P(1)=\frac{5!}{(5-1)!1!} \times .3^{1} \times .7^{4}=.36015 \\
& P(0)=\frac{5!}{(5-0)!0!} \times .3^{0} \times .7^{5}=.16807
\end{aligned}
$$

$$
P(\geq 2)=.3087+.36015+.16807=0.83692
$$

## Chapter 6:

## Standard Normal Z Score Probabilities

Find the probability that a $z$-score is between -1.5 and 2 . That is, find $P(-1.5 \leq z \leq 2)$
Note: $\mu=0$ because we are talking Standard Normal Distribution $\sigma=1$ because we are talking Standard Normal Distribution
$z=\frac{x-u}{\sigma}$ standard score $z=\frac{-1.5-0}{1}=-1.5 \quad z=\frac{2-0}{1}=2.0$
Probability for $Z=-1.5$ is .0668 Probability for $Z=2.0$ is $.9772 \quad P(-1.5 \leq z \leq 2)=.9104$

## IQs Z Score Probabilities

Adult IQs are normally distributed with $\mu=100$ and $\sigma=15$. Find the probability that a randomly selected IQ is less than 112. That is, find $P(x<112)$.
$z=\frac{x-u}{\sigma}$ standard score $z=\frac{112-100}{15}=0.8$ The probability of a $Z$ score of 0.8 is .7881

## IQs Z Score Probabilities

Adult IQs are normally distributed with $\mu=100$ and $\sigma=15$. Find the probability that a randomly selected $I Q$ is at least 122. That is, find $P(x \geq 122)$.
$z=\frac{x-u}{\sigma}$ standard score $z=\frac{122-100}{15}=1.47$ The probability of a $Z$ score of 1.47 is .9292
Because we need at least, the $P=1-.9292=.0708$

## IQs Z Score Probabilities

Adult IQs are normally distributed with $\mu=100$ and $\sigma=15$. Find the probability that a randomly selected IQ is between 112 and 122. That is, find $P(112 \leq x \leq 122)$.
$z=\frac{x-u}{\sigma}=\frac{122-100}{15}=1.47$ The probability of a $Z$ score of 1.47 is .9292
$z=\frac{x-u}{\sigma}=\frac{112-100}{15}=0.8$ The probability of a $Z$ score of 0.8 is 7881
$P(112 \leq x \leq 122)=.9292-.7881=.1441$

## IQs Z Score Sample Probabilities

Adult IQs are normally distributed with $\mu=100$ and $\sigma=15$. Find the probability if 9 students are randomly selected that one of those selected has an IQ of at least 122. That is, find $P$ ( $x$ $\geq 122$ ).

$$
\begin{aligned}
& \sigma_{\bar{x}}=\frac{\sigma}{\sqrt{n}} \text { central limit theorem (standard error) } \sigma_{\bar{x}}=\frac{15}{\sqrt{9}}=5 z=\frac{x-u}{\sigma_{x}} \\
& z=\frac{122-100}{5}=4.4 \text { Probability of } .9999 \\
& P(x \geq 122)=1-.9999=.0001
\end{aligned}
$$

## Probability (finding a Z-score given a shaded area)

Given the shaded area is .75. Find the z-score for this area using Positive $Z$ score chart or technology. $Z$ chart shows a $Z$ score of 0.67 for a probability of .75


## Probability (finding a Z-score given a shaded area)

Given the shaded area is .75. Find the $z$-score for area to the left of this shaded area using Z score chart or technology.

Subtract . 75 from 1 to get the unshaded area. 1-. $75=.25$
$Z$ chart shows a $Z$ score of -0.67 for a probability of .25


## Probability (finding a IQ-score given a shaded area)

Adult IQs are normally distributed with $\mu=100$ and $\sigma=15$. Find the IQ scores for those individuals whose IQ exceed $75 \%$ of the population. $Z$ chart shows a $Z$ score of 0.67 for a probability of $\mathbf{7 5}$

$$
z=\frac{x-u}{\sigma} \quad .67=\frac{x-100}{15} \quad .67(15)=\mathrm{x}-100 \quad \mathrm{x}=110.5 \text { Answer } \mathrm{x}>\mathrm{IQ} \text { score of } 110 .
$$

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## Chapter 7

## Finding the "Critical Value" for a specific "Confidence Level"

Example: Find the "Critical Value" for a "Confidence Level of $90 \%$ ".
$z_{\alpha / 2}$ is also known as the critical value for two tailed data.

1. $\alpha=1-$ confidencelevel $=1-.90=.10$
2. $\frac{\alpha}{2}=\frac{.10}{2}=.05$
3. $1-.05=.95$
4. After looking up a . 95 Confidence level on the Z-score chart (Table A-2), the Critical value Z-score was determined to be 1.645

Therefore the critical value of a $90 \%$ Confidence Level is 1.645

The table below was developed using the procedure outlined above.

| Confidence <br> Level | Critical <br> Value <br> (Z-score) |
| :--- | :--- |
| 0.90 | 1.645 |
| 0.91 | 1.70 |
| 0.92 | 1.75 |
| 0.93 | 1.81 |
| 0.94 | 1.88 |
| 0.95 | 1.96 |
| 0.96 | 2.05 |
| 0.97 | 2.17 |
| 0.98 | 2.33 |
| 0.99 | 2.575 |

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## Finding Confidence Intervals "E" for a Proportion

Sample: A research poll 200 respondents were asked if they felt vulnerable to identity theft. Thirty said yes. Find the confidence interval given a confidence level of $95 \%$.
$\mathrm{N}=200 \mathrm{x}=30$ confidence interval of $95 \% z_{\alpha / 2}=1.96 \quad \mathrm{p}=\frac{30}{200}=.15 \quad \mathrm{q}=1-.15=.85$

$$
E=z_{\alpha / 2} \sqrt{\frac{\hat{p} \hat{q}}{n}}=1.96 \sqrt{\frac{.15 \times .85}{200}}=0.4949
$$

## To find a confidence interval for Mean (known $\sigma$ )

$$
E=z_{\alpha / 2} \frac{\sigma}{\sqrt{n}}
$$

Example: A sample of 5 students were asked if they felt their test was fair. Three said yes. Find the confidence interval given a confidence level of $95 \%$ and a $\sigma$ of 5 .

$$
\sigma=5, \text { mean }=3, n=5, \text { Confidence level }=.95 \quad z_{\alpha / 2}=1.96
$$

$$
E=z_{\alpha / 2} \frac{\sigma}{\sqrt{n}}=1.96 \frac{5}{\sqrt{5}}=4.3827=4.383
$$

## To find a confidence interval for Mean (unknown $\sigma$ )

$$
E=t_{\alpha / 2} \frac{s}{\sqrt{n}}
$$

Example: Find the confidence interval for a sample of 4 students given a sample mean of 6.8 , a sample standard deviation of 1 and a confidence level of $95 \%$

$$
\begin{aligned}
& t_{\alpha / 2}=3.182 \quad \text { from T Chart for df } 3 \text { for two tail } .05 \\
& E=t_{\alpha / 2} \frac{s}{\sqrt{n}}=3.182 \frac{1}{\sqrt{4}}=1.591
\end{aligned}
$$

## Determine Minimum sample size

## Proportion ( $\hat{p}$ and $\hat{q}$ are known)

Sample question: $41 \%$ of Jacksonville residents said that they had been in a hurricane. How many adults should be surveyed to estimate the true proportion of adults who have been in a hurricane, with a $95 \%$ confidence level and an interval of $3 \%$ points?
$n=\frac{\left[z_{\alpha / 2}\right]^{2} \hat{p} \hat{q}}{E^{2}} \quad$ proportion $(\hat{p}$ and $\hat{q}$ are known $)$

$$
n=\frac{[1.96]^{2}(.41)(.59)^{\wedge}}{.03^{2}}=1032.536711=1032.536711=1033
$$

## Proportion ( $\hat{p}$ and $\hat{q}$ are not known)

Sample question: Jacksonville residents said that they had been in a hurricane. How many adults should be surveyed to estimate the true proportion of adults who have been in a hurricane, with a $95 \%$ confidence level and an interval of $3 \%$ points?
$n=\frac{\left[z_{\alpha / 2}\right]^{2} \cdot 0.25}{E^{2}} \quad$ proportion $(\hat{p}$ and $\hat{q}$ are not known $)$
$n=\frac{[1.96]^{2}(.25)^{\wedge}}{.03^{2}}=1067.1111=1067.1111=1068$
Note: When finding the minimum sample size, always round up to whole number.

## Proportion (known standard deviation of the population $\sigma$ )

Sample question: Suppose we want to know the average age of a Florida State College student, plus or minus 0.5 years. We'd like to be $99 \%$ confident about our result. From a previous study, we know that the standard deviation for the population $\sigma$ is 2.9.

$$
n=\left[\frac{z_{\alpha / 2} \sigma}{E}\right]^{2}
$$

$n=\left[\frac{2.575(2.9)}{, 5}\right]^{2}=223.054225=223.054225=224$
Note: When finding the minimum sample size, always round up to whole number.

## Chapter 8:

## Statistical Hypotheses

The best way to determine whether a statistical hypothesis is true would be to examine the entire population. Since that is often impractical, researchers typically examine a random sample from the population. If sample data are not consistent with the statistical hypothesis, the hypothesis is rejected.

There are two types of statistical hypotheses.

- Null hypothesis. The null hypothesis, denoted by $\boldsymbol{H}_{0}$, is usually the hypothesis that sample observations result purely from chance.
$\checkmark H_{0}$ is always $=$.
- Alternative hypothesis. The alternative hypothesis, denoted by $\boldsymbol{H}_{1}$, is the hypothesis that sample observations are influenced by some non-random cause.
$\checkmark \boldsymbol{H}_{1}$, is never $=$. Options are $<,>, \neq$.


## Chart for establishing $H_{0}: P$ and $H_{1}: P$

| Claim | $\mathrm{H}_{0}: \mathrm{P}$ | $\mathrm{H}_{1}: \mathrm{P}$ | Comments |
| :--- | :--- | :--- | :--- |
| $\mathrm{P}>5$ | $\mathrm{H}_{0}: \mathrm{P}=5$ | $\mathrm{H}_{1}: \mathrm{P}>5$ |  |
| $\mathrm{P}<5$ | $\mathrm{H}_{0}: \mathrm{P}=5$ | $\mathrm{H}_{1}: \mathrm{P}<5$ |  |
| $\mathrm{P}=5$ | $\mathrm{H}_{0}: \mathrm{P}=5$ | $\mathrm{H}_{1}: \mathrm{P} \neq 5$ | If difference between the assumption and the claim is close. <br> For instance, Claim 20\% of adults smoke. Survey showed $21 \%$ <br> of respondents smoke. |
| $\mathrm{P} \neq 5$ | $\mathrm{H}_{0}: \mathrm{P}=5$ | $\mathrm{H}_{1}: \mathrm{P} \neq 5$ | If difference between the assumption and the claim is close. <br> For instance, Claim $20 \%$ of adults smoke. Survey showed $21 \%$ <br> of respondents smoke. |
| $\mathrm{P} \geq 5$ | $\mathrm{H}_{0}: \mathrm{P}=5$ | $\mathrm{H}_{1}: \mathrm{P}<5$ | $\mathrm{H}_{1}$ is opposite the claim |
| $\mathrm{P} \leq 5$ | $\mathrm{H}_{0}: \mathrm{P}=5$ | $\mathrm{H}_{1}: \mathrm{P}>5$ | $\mathrm{H}_{1}$ is opposite the claim |

## Types of Hypothesis Tests - P Value One tail vs two tail

$\mathrm{H}_{1}$ : P < Left Tailed test Actual P value
$H_{1}: P>$ Right-tailed test 1 -actual $P$ value
$H_{1}: P \neq$ Two tailed test. If test is a 2 tail, you have to double your calculated $P$ value

## Acceptance/Rejection

| Condition | Conclusion |
| :--- | :--- |
| Original claim does not include equality and you <br> reject $H_{0}$ because the $p$ value is $<$ or $=$ to given <br> Critical value $\alpha$. | There is sufficient evidence to support the original <br> claim |
| Original claim does not include equality and you <br> fail to reject $H_{0}$ because the $p$ value is > than the <br> given Critical value $\alpha$. | There is not sufficient evidence to support the <br> original claim |
| Original claim includes equality and you reject $H_{0}$ <br> because the $p$ value is < or $=$ to the given Critical <br> value $\alpha$. | There is sufficient evidence to warrant rejection of <br> the original claim |
| Original claim includes equality and you fail to <br> reject $H_{0}$ because the $p$ value is > than the given <br> Critical value $\alpha$. | There is not sufficient evidence to warrant <br> rejection of the original claim |

# Hypothesis Tests for Proportions (Test Statistic, P value and sample proportion). 

$$
z=\frac{p-p}{\sqrt{\frac{p q}{n}}}
$$

Example: In sampling 200 people, we found that $30 \%$ of them favored a certain candy. Use $\alpha=$ $10 \%$ to test the hypothesis that the proportion of people who favored that candy is less than $35 \%$.

Solution: This represents a one-sample test of proportion. So we use the "1-PropZTest" function. The sample proportion is $30 \%$ or $p=0.30$, and the hypothesized claim is $p \geq 0.35$. Therefore, the hypotheses are $\mathrm{H}_{0}: p=0.35$ and $\mathrm{H}_{1}: p<0.35$. This is a two tail test because the clam includes an equal sign.
[NOTE: If the $p$-value $<\alpha$, reject the null hypothesis; otherwise, do not reject the null hypothesis].

$$
z=\frac{p-p}{\sqrt{\frac{p q}{n}}}=\frac{.30-.35}{\sqrt{(.35)(.65) / 200}}=-1.48 \quad \mathrm{P}=.0694
$$

This is a left tail test because $\boldsymbol{H}_{1}$ : $\boldsymbol{P}$ < so $\mathrm{P}=.0694$
P > than a so fail to reject the null hypothesis

## Hypothesis Tests for Proportions for a Mean (known $\sigma$ )

$$
z=\frac{\bar{x}-u}{\sigma / \sqrt{n}}
$$

Example: A sample of size 200 has a mean of 20 and a standard deviation of 16 and $\sigma=6$. Test the hypothesis claim that the population mean is not equal to19.2 ( $\alpha=10 \%$ level of significance).

$$
z=\frac{\bar{x}-u}{\sigma / \sqrt{n}}=\frac{19.2-20}{6 / \sqrt{200}}=-1.8856 \quad \mathrm{P}=.02968 \text { from chart }
$$

This is a two tail test $\left(\mathrm{H}_{1}: \mathrm{P} \neq\right)$ so $\mathrm{P}=2 \mathrm{x} .02968=.05936$

## P < than a so reject the null hypothesis

## Using the T-Chart

Example: A researcher claims that the mean speed of a Miss Williams tennis serve is 125 mph .
Therefore, $H_{0}=125$ and the alternate hypothesis is $H_{1} \neq 125$. Assume that data is normally distrusted for a sample test of 40 serves and the value of a is unknown. Find the critical values for the hypothesis test with a significance level of 0.05 . This is a two tail test because equal is included in the claim.

Go to the Table 3 Distribution Critical t values. Look in the row for a degree of freedom (df) of 39 (40-1) and the column for two tail significance level of 0.05 . The answer is $\pm 2.023$.

## This is a two tail.

Under two tail the values are:

| Two Tail | 0.01 | 0.02 | 0.05 | 0.10 | 0.20 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| df 39 | 2.708 | 2.426 | $\mathbf{2 . 0 2 3}$ | 1.685 | 1.304 |

In the row for degree of freedom of 39, the number on the line for two tail of 0.05 .
Example: A researcher claims that the mean speed of a Miss Williams tennis serve is $>125 \mathrm{mph}$. Therefore, $H_{0}=125$ and the alternate hypothesis is $H_{1}>125$. Assume that data is normally distrusted for a sample test of 40 serves and the value of $\alpha$ is unknown. Find the critical values for the hypothesis test with a significance level of 0.05 . This is a one tail test because equal is not included in the claim.

Go to the Table 3 Distribution Critical $t$ values. Look in the row for df is 39 (40-1) and the column for one tail significance level of 0.05. The answer is 1.685 .

Under one tail the values are:

| OneTail | 0.005 | 0.01 | 0.025 | 0.05 | 0.10 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| df 39 | 2.708 | 2.426 | 2.023 | 1.685 | 1.304 |

## The P-value for a right-tailed test when the given test statistic is not on the chart.

Example with $\mathrm{n}=28$ and test statistic $\mathrm{t}=3.026$.
The degree of freedom is $28-1=27$. This is a one tail test.
On the t -Distribution chart go to the row for degree of freedom of 27, the highest number on this line, 2.771 corresponds to a one tail of 0.005 .

| One Tail | 0.005 | 0.01 | 0.025 | 0.05 | 0.10 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| df 27 | 2.771 | 2.473 | 2.052 | 1.703 | 1.314 |

The $t=3.026$ given in the problem is higher than the highest value of 2.771 for .005 , therefore the probability must be less than 0.005 .

## Hypothesis Tests for a Mean (unknown $\sigma$ )

## Find test statistic and $p$ value of a T-Test

Example: A sample of size 20 has a mean of 110 and a standard deviation of 16. Use the TI-83 calculator to test the hypothesis claim that the population mean is greater than 100 with a level of significance of $\alpha=5 \%$.

$$
t=\frac{\bar{x}-u}{s / \sqrt{n}}=\frac{110-100}{16 / \sqrt{20}}=2.7951
$$

This value falls between .005 and .01 for a one tail with a df of 19 .

Original hypothesis claim that the population mean is greater than 100 does not include equality and you reject $H_{0}$ because the $p$ value is <or $=$ to given Critical value $\alpha$. There is sufficient evidence to support the original claim.

