## Statistics Tutorial, Formula Applications and TI-84 Instructions



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## Chapter 1

## Statistical Literacy and Critical Thinking

## Parameter and Statistic "What is a parameter, and what is a statistic?

A parameter is a numerical measurement describing some characteristic of a population. (p with $p$ )
A statistic is a numerical measurement describing some characteristics of a sample. (s with s)

## How do quantitative data and categorical data differ?

Quantitative data consists of numbers representing counts or measurement.
Categorical data can be separated into different categories' that are distinguished by some characteristic that is not numerical.

## How do discrete and continuous data differ?

The numbers of people surveyed in each of the next several Gallup polls
Discrete because it is quantitative and the answer will be countable
The exact heights of individuals in a sample of several statistics students
Continuous because there can be numerous results. Heights can be anywhere from 1 to 8 feet
The number of Super Bowl football games that must be played before one of the teams scores exactly 75 points

Discrete because it is quantitative.

## Identifying the Population

In a Gallup poll of 1010 adults in the United States, $55 \%$ of the respondents said that they used local TV stations daily as a source of news. Is the 1010 value a statistic or a parameter? Is the $55 \%$ value a statistic or a parameter? Describe the population.

The values of 1010 and $55 \%$ are both statistics because they are based on a sample.
The population would consist of all adults in the United States.

## Chapter 2 and 3:

## EDIT Feature using the Calculator

EDIT-1: Edit... allows the entry (in columns) and modifications of data to be used with the CALC and TESTS features.

## Press STAT, EDIT,1: Edit

Press ENTER, and you may delete, enter or modify data that is already in the list columns.


## Clearing lists

Place cursor on top of the list name (L1, etc.) at the top of the column. Press the CLEAR key, then press ENTER. When you press the Down arrow all existing data will be removed from that column.


## Changing existing entry

Change the 18 to 20. Press STAT, 1: Edit, ENTER,
Move down arrow to the highlight 20, type18, and hit ENTER.
The 18 will replace the 20 .

## Return to Home Screen

Press 2nd MODE [QUIT] to end the EDIT mode and return to the home screen


## Descriptive Statistics for one set of Raw Data

Find the midrange, mean, median, and mode for the given sample data.
Listed below are the ages of motorcyclists when they were fatally injured in 18 traffic crashes.


## Using the Calculator

## Given 5 numbers determine the mean, median, and mode

Press STAT, EDIT, 1: Edit,
Press ENTER
EDIT CALC TESTS

1. Edit

Clear the home screen. (See Edit instructions)
Enter all numbers into L1. Press ENTER after each entry until all the numbers are inputted.
$(10,12,13,20,18)$

Once all data is entered. Press STAT, right arrow to CALC.

## 1: 1-Var Stats

Press ENTER.
L1 appears in List. If another List is to be used enter the list number. FreqList should be blank

Use the down arrow to scroll down to Calculate

Press ENTER for the results.

Press the down arrow to see the remainder of the output.



EDIT CALC TESTS 1. 1 Var Stats

1-Var St.at.s
List: L1
FreqList: blank Calculate:

> 1-Vars Stats
> $\bar{x}=14.6$
> $\sum x=73$
> $\sum x^{2}=1137$
> $S x=4.219004622$
> $\sum x=3.77592453$
> $n=5$
$n=5$
$\operatorname{Min} X=10$
Q1=11
Med-13
Q3 = 19
MaxX=20

## Summary Statistics for one set of Raw Data using Frequency Distribution

## Statistical Literacy and Critical Thinking

A set of data has a maximum value of 74 , a median value of 61 , and a minimum value of 48 .

Use the range rule of thumb to estimate the standard deviation, rounding to the nearest tenth.

$$
s=\frac{\text { range }}{4}=\frac{74-61}{4}=\frac{13}{4}=3.25=3.3
$$

## Analysis of Frequency Distribution

Identify, the class width, class midpoints and class boundaries.

| Age <br> (years) | Frequency | Mid <br> points | Class <br> boundaries |
| :--- | ---: | ---: | ---: |
| $9-11$ | 3 | 10 | $8.5-11.5$ |
| $12-14$ | 2 | 13 | $11.5-14.5$ |
| $15-17$ | 5 | 16 | $14.5-17.5$ |
| $18-20$ | 3 | 19 | $17.5-20.5$ |
| $21-23$ | 6 | 22 | $20.5-23.5$ |
| $24-27$ | 4 | 25 | $23.5-26.5$ |

Class width = 3 (12-9)
The difference between the first numbers of two consecutive class intervals.

Class midpoints $=\frac{11+9}{2}=10$
The midpoint of a class is the sum of the first and last number in the class divided by 2.

## Class boundaries

The sum of the last number and first number of two consecutive class intervals divided by 2 . $\frac{11+12}{2}=11.5$

Is it possible to identify the actual age of the 5 people in age from 15-17 years?
No! The upper and lower limits include the specific ages. You know 5 people have ages between 15-17 years, but you do not know their actual age.

## Using the Calculator

Given 6 class midpoints and there related frequencies, determine the mean, median, and mode.

Press STAT, EDIT, 1: Edit,

Press ENTER,
EDIT CALC TESTS

1. Edit

Clear the home screen (see EDIT instructions)
Enter the class midpoints of your data into L1 (10, 13, 16, 19, 22, 25).

Enter the frequencies of your class
midpoints into $\mathrm{L} 2(3,2,5,3,6,4)$.
Once all data is entered. Press STAT, right arrow to CALC.

1: 1-Var Stats

L1 appears in List.
FreqList should L2
Use the down arrow to scroll down to Calculate
Press ENTER for results

Press the down arrow to see the remainder of the output.

## Chapter 4:

## Basic Concepts of Probability

## Criteria for a Probability

Which of the following values may NOT be a probability?
a) $\frac{1}{3}$
b) -1
c. 0.87
d) 1.01
e) $75 \%$
$b$ and d Probabilities cannot be minus or $>1$

## Relative Frequency Probability

If a person is randomly selected, find the probability that his or her birthday is October 17. Ignore leap years.

$$
P(A)=\frac{\text { number of times } A \text { occurred }}{\text { number of times trial was repeated }}=1 / 365
$$

If a person is randomly selected, find the probability that his or her birthday is in October.

$$
P(A)=\frac{\text { number of times } A \text { occurred }}{\text { number of times trial was repeated }}=31 / 365
$$

## Multiplication Rule

e set of digits $\{0,1,2,3,4,5,6,7\}$ will be used to create a five-digit code. How many five-digit codes can be created if...
a. there are no restrictions? $\mathbf{8}^{5}=\mathbf{3 2 7 6 8}$
b. the digits may not be repeated? $\mathbf{8} \mathbf{X 7} \mathbf{7} \mathbf{6} \mathbf{X} \mathbf{5 X} \mathbf{4}=\mathbf{6 7 2 0}$
c. the code must begin with a 3 and the digits may be repeated? $8^{4}=4096$

Two different cards are drawn from a standard deck of 52 cards. What is the probability that the first card is a king and the second card is a queen?

$$
\frac{4}{52} \times \frac{4}{51}=\frac{16}{2652}=.00603
$$

A box of marbles has 3 yellow marbles, 2 green marbles, and 5 red marbles. Suppose that two marbles are selected without replacement. What is the probability that neither selected marble is green?

$$
\frac{8}{10} \times \frac{7}{9}=\frac{56}{90}=.6222
$$

## Factorial n!

Find the value of $5!=5 \times 4 \times 3 \times 2 \times 1=120$

## Using the calculator

Math Num CPX PRE
1: $r$ and
2: ${ }_{n} P_{r}$
3: ${ }_{n} C_{r}$
4: !
First enter " n " (5), then select MATH->PRB-then Scroll down to! and hit ENTER

Then hit ENTER again and the answer 120 appears

$$
5!\quad 120
$$

## Permutations nPr

If a president and vice president are to be selected from 5 committee members how many permutations can be selected In this example order is important so we use Permutations ${ }_{n} P_{r}$

$$
{ }_{5} P_{2}=\frac{5!}{(5-2)!}=\frac{5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1}=\frac{120}{6}=20
$$

## Using the calculator

First enter "n"(5), then select MATH->PRB-then Scroll down to nPr and hit ENTER

Press 2 and hit ENTER again and the answer 20 appears

Math Num CPX PRE
1: $r$ and
2: ${ }_{n} P_{r}$

## Combinations ${ }_{n} C_{r}$

If 3 committee members are select from 5 employees how many combinations can be selected. In this example order is not important so we use Combinations $={ }_{n} C_{r}$
${ }_{5} C_{3}=\frac{5!}{(5-3)!(3!}=\frac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 1 \times 3 \times 2 \times 1}=\frac{120}{12}=10$

Math Num CPX PRE
1: $r$ and
2: ${ }_{n} P_{r}$
3: ${ }_{n} C_{r}$

Using the calculator
First enter "n"(5), then select MATH->PRB-then Scroll down to ${ }_{n} C_{r}$ and hit ENTER

Press 3 and hit ENTER again and the answer 10 appears
${ }_{5} C_{3}$
ANS 10

## Statistical Thinking

During a certain week the number of late and on-time flights were counted for three airlines.

|  | Number of <br> On-time Flights | Number of <br> Late Flights | Total |
| :--- | ---: | :--- | :--- |
| Southern Airlines | 34 | 11 | 45 |
| Statewide Airlines | 28 | 3 | 31 |
| Westward Airlines | 30 | 8 | 38 |
| Total | 92 | 22 | 114 |

What is the probability that the flight was a Westward Airlines flight and was late?

$$
\frac{8}{114}=.0702
$$

What is the probability that the flight was late, given that the flight was on Westward Airlines?

$$
\frac{8}{38}=.2105
$$

An experiment has a sample space given by $S=\{1,2,3,4,5,6,7,8,9,10\}$. The events $D, E$, and $F$ are defined as $D=\{1,2,3,4\}, E=\{2,4,6,8,10\}$, and $F=\{3,5,7\}$.

Assume each outcome is equally likely:
Are events D and E disjoint (mutually exclusive)? No
Are events D and F disjoint (mutually exclusive)? No
Are events E and F disjoint (mutually exclusive)? Yes
Determine whether events G and H are independent or dependent.
G: Joe forgets to set his alarm clock. H: Joe is late for work the next day. Dependent
G: Milk is on sale. $\quad \mathrm{H}$ : Notebook paper is on sale. Independent
G: A student earns an "A" in English 1.
H: A student in English 1 is a girl. Independent

## Calculating Odds

A box contains 6 white chips, 3 blue chips, 2 yellow chips, and 1 green chip. A player randomly selects one chip from the box. If the player selects a blue chip, he/she wins $\$ 3$. If the player selects any other color chip, he/she loses $\$ 1$

Find the players odds of winning?

$$
\text { odds }=\frac{\# \text { favorable }}{\# \text { unfavorable }}=\frac{3}{9}=\frac{1}{3} \quad 1: 3
$$

Find the player's expected value?

$$
u=\sum x \bullet P(x)=3\left(\frac{3}{12}\right)+-1\left(\frac{9}{12}\right)=0
$$

When you give a casino $\$ 5$ for a bet on the "pass the line" in the game of craps, there is a $244 / 495$ probability that you will win $\$ 5$ and a 251/495 probability that you will lose $\$ 5$.

What are your odds of winning? $\quad$ odds $=\frac{\# \text { favorable }}{\# \text { unfavorable }}=244: 251$

What is your expected value?

$$
u=\sum x \bullet P(x)=5\left(\frac{244}{295}\right)+-5\left(\frac{251}{495}\right)=-.0 .0707 \text { cents }
$$

In the long run, how much do you lose for each dollar bet? $\frac{-0.0707}{5}=-0.014=1.4$ cents

## Chapter 5:

## Parameters for Binomial Distribution

Among 945 births, there were 879 girls born to parents. Assuming boys and girls are equally likely, find the mean and standard deviation for the number of girls born in this group of 945 babies.
$u=n \bullet p \quad$ mean binomial
$\sigma=\sqrt{n \bullet p \bullet q}$ standard deviation binomial
$\sigma^{2}=n \bullet p \bullet q$ variance binomial

For this example, $n=945$ and $p=0.50 q=0.50$
Mean: $\quad u=n \bullet p \quad u=945 \bullet 0.50=472.5$ girls
Standard Deviation: $\quad \sigma=\sqrt{945 \bullet .5 \bullet .5}=15.3704=15.4$ girls
variance binomial $\sigma^{2}=n \bullet p \bullet q=945 \bullet 0.50 \bullet 0.50=236.25$
Find the minimum usual number and maximum usual number of girls born in this group of 945. (referred to as Rule of Thumb)

Minimum usual value: $\mu-2 \sigma=472.5-2(15.4)=441.7$ girls
Maximum usual value: $\mu+2 \sigma=472.5+2(15.4)=503.3$ girls

## Binomial Probability Distribution Values

Find the probability that exactly 2 successes will occur out of 5 trials if the probability of success is 0.3 .

$$
\begin{aligned}
& P(x)=\frac{n!}{(n-x)!x!} \bullet p^{x} q^{n-x} \text { binomial probability } \\
& Q=1.0-p=1.0-.3=.7 \\
& P(2)=\frac{5!}{(5-2)!2!} \bullet .3^{2} .7^{3}=.3087
\end{aligned}
$$

| Using the calculator: When term is exactly! |  |
| :---: | :---: |
| Press $\mathbf{2 n d}^{\text {nd }}$ VARS [DISTR]. | DISTR Draw <br> A. binompdf( |
| Scroll down to A:binompdf( Press ENTER |  |
| Trials $=5$ | Binormalcdf Trials |
| $\mathrm{P}=.30$ |  |
| $x$ Value $=2$ | x:value: Paste: |
| Enter $\mathrm{n}, \mathrm{p}, \mathrm{x}$-value) $\mathrm{n}=5, \mathrm{p}=0.3, \mathrm{x}=2$ and scroll down to Paste | Binompdf (5, .3, 2) |
| and press ENTER. Press ENTER again to get the answer. | . 3087 |

Find the probability that 2 or less successes will occur out of 5 trials if the probability of success is 0.3 . That is, find $P(x \leq 2)$.

Press $2^{\text {nd }}$ VARS [DISTR]. When term is less than or more than

Scroll down to B:binomcdf(
Press ENTER.

| Trials $=5$ | Binormalcdf <br> Trials <br> P: <br> X:value: <br> Paste: |
| :--- | :--- |

Enter $\mathrm{n}, \mathrm{p}, \mathrm{x}$-value) $\mathrm{n}=5, \mathrm{p}=\mathbf{0 . 3}, \mathrm{x}=2$ largest \# scroll down to
Paste and press ENTER. Press ENTER again to get the answer

Binomcdf (5,.3,2)
DISTR Draw
B. binomcdf(

Binormalcdf
Trials
x:value:
Paste:
.83692

## Chapter 6:

## Standard Normal Z Score Probabilities

Find the probability that a $z$-score is between -1.5 and 2 . That is, find $P(-1.5 \leq z \leq 2)$
Note: $\mu=0$ because we are talking Standard Normal Distribution
$\sigma=1$ because we are talking Standard Normal Distribution
$z=\frac{x-u}{\sigma}$ standard score $z=\frac{-1.5-0}{1}=-1.5 \quad z=\frac{2-0}{1}=2.0$
Probability for $Z=-1.5$ is .0668 Probability for $Z=2.0$ is $.9772 \quad P(-1.5 \leq z \leq 2)=.9104$

| Using the Calculator |  |
| :---: | :---: |
| Press $2^{\text {nd }}$ VARS [DISTR]. | DISTR Draw <br> 2. normalcdf( |
| Scroll down to 2: normal cdf Press ENTER |  |
| Lower $=-1.5$ | Normalcdf |
|  | Lower: |
| Upper $=2.0$ | Upper: |
|  | $\mu=0$ |
| $\mu=0$ | $\sigma=1$ |
|  | Paste: |
| $\sigma=1$ |  |
| Enter Lower Z score, Upper Z score scroll down to Paste and press ENTER. | nomalcdf $(-1.5,2,0,1)$ .9104427093 |
| Press ENTER again to get the answer .91044. |  |

## IQs Z Score Probabilities

Adult IQs are normally distributed with $\mu=100$ and $\sigma=15$. Find the probability that a randomly selected $I Q$ is less than 112. That is, find $P(x<112)$.

$$
z=\frac{x-u}{\sigma} \text { standard score } z=\frac{112-100}{15}=0.8 \text { The probability of a } Z \text { score of } 0.8 \text { is } .7881
$$



## IQs Z Score Probabilities

Adult IQs are normally distributed with $\mu=100$ and $\sigma=15$. Find the probability that a randomly selected $I Q$ is between 112 and 122. That is, find $P(112 \leq x \leq 122)$.
$z=\frac{x-u}{\sigma}=\frac{122-100}{15}=1.47$ The probability of a $Z$ score of 1.47 is .9292
$z=\frac{x-u}{\sigma}=\frac{112-100}{15}=0.8$ The probability of a $Z$ score of 0.8 is 7881
$P(112 \leq x \leq 122)=.9292-.7881=.1441$

| Using the Calculator |  |
| :---: | :---: |
| Press $2^{\text {nd }}$ VARS [DISTR]. <br> Scroll down to 2: normal cdf Press ENTER | DISTR Draw <br> 2. normalcdf( |
| $\begin{aligned} & \text { Lower = } 112 \\ & \text { Upper }=122 \\ & \mu=100 \\ & \sigma=15 \end{aligned}$ | Normalcdf <br> Lower: <br> Upper: $\mu=0$ $\sigma=1$ <br> Paste: |
| The sequence is normalcdf (lower, upper, $\boldsymbol{\mu}, \boldsymbol{\sigma}$ ). <br> Enter 112,122,100,15)scroll down to Paste and press ENTER <br> Press ENTER again to get the answer . 1406 . | normalcdf $(112,122,100,15)$ <br> .1406219197 |

## IQs Z Score Sample Probabilities

Adult IQs are normally distributed with $\mu=100$ and $\sigma=15$. Find the probability if 9 students are randomly selected that one of those selected has an IQ of at least 122. That is, find P (x $\geq$ 122).

$$
\begin{array}{ll}
\sigma_{\bar{x}}=\frac{\sigma}{\sqrt{n}} \text { central limit theorem (standard error) } & \sigma_{\bar{x}}=\frac{15}{\sqrt{9}}=5 z=\frac{x-u}{\sigma_{x}} \\
z=\frac{122-100}{5}=4.4 \text { Probability of } .9999 & P(x \geq 122)=1-.9999=.0001
\end{array}
$$



## Probability (finding a Z-score given a shaded area)

Given the shaded area is .75. Find the $z$-score for this area using Positive $Z$ score chart or technology. $Z$ chart shows a $Z$ score of 0.67 for a probability of .75


## Using the Calculator

DISTE DRFW
1: normalpdf(
2: nomalodf(
Ginutorms
5:invot
5: t.pdf
G:t.edf

Scroll down to 3:invNorm Press ENTER.
Area: . 25
$\mu=0$
$\sigma=1$
Enter 25 scroll down to Paste and press ENTER. Press ENTER
again to get the answer $\mathrm{Z}=-.675$ again to get the answer $\mathrm{Z}=-. .675$

InvNorm (.25)
$-.6744897495$

Probability (finding a Z-score given a shaded area)
Given the shaded area is .75. Find the z-score for area to the left of this shaded area using Z score chart or technology.

Subtract . 75 from 1 to get the unshaded area. 1-. $75=.25$
$Z$ chart shows a $Z$ score of -0.67 for a probability of .25


Using the Calculator
Press $2^{\text {nd }}$ VARS [DISTR].
Scroll down to 3:invNorm Press ENTER.
Area: . 25
$\mu=0$
$\sigma=1$
DISTE DRFW
I: normelpdfs 2: normaledf?
名 inu Horm?
Sinutor
5: $\operatorname{ind}_{\text {tedf }}$ 6: tedf
$74 \times 2 \mathrm{Fdf}$
invnorm
Area:
$\mu=0$
$\sigma=1$
Paste:
Enter .75 scroll down to Paste and press ENTER .
Press ENTER again to get the answer $Z=-.674$

InvNorm (.25)
-. 6744897495

## Probability (finding a Z-score given a shaded area)

Adult IQs are normally distributed with $\mu=100$ and $\sigma=15$. Find the IQ scores for those individuals whose IQ exceed $75 \%$ of the population. $Z$ chart shows a $Z$ score of -0.67 for a probability of . 25

$$
z=\frac{x-u}{\sigma} \quad .67=\frac{x-100}{15} \quad .67(15)=x-100 \quad x=110.05
$$



| Using the Calculator <br> Press $2^{\text {nd }}$ VARS [DISTR]. <br> Scroll down to 3:invNorm <br> Press ENTER. |  |
| :---: | :---: |
| Area: . 75 $\begin{aligned} & \mu=0 \\ & \sigma=1 \end{aligned}$ | invnorm <br> Area: $\begin{gathered} \mu=0 \\ \sigma=1 \end{gathered}$ Paste: |
| Enter .75 scroll down to Paste and press ENTER . <br> Press ENTER again to get the answer $\mathrm{Z}=. \mathbf{. 6 7}$ | $\begin{aligned} & \text { InvNorm (.75) } \\ & .6744897495 \end{aligned}$ |
| The IQ score for those that exceed $75 \%$ of the population is equal to: $I Q>100+(.67 \times 15) I Q>110.05$ |  |

## Chapter 7

## Finding the "Critical Value" for a specific "Confidence Level"

## Example: Find the "Critical Value" for a "Confidence Level of 90\%".

$z_{\alpha / 2}$ is also known as the critical value for two tailed data.

1. $\alpha=1-$ confidencelevel $=1-.90=.10$
2. $\frac{\alpha}{2}=\frac{.10}{2}=.05$
3. $1-.05=.95$
4. After looking up a . 95 Confidence level on the Z-score chart (Table A-2), the Critical value Z-score was determined to be 1.645

Therefore the critical value of a 90\% Confidence Level is 1.645

The table below was developed using the procedure outlined above.

| Confidence <br> Level | Critical <br> Value <br> (Z-score) |
| :--- | :--- |
| 0.80 | 1.28 |
| 0.85 | 1.44 |
| 0.90 | 1.645 |
| 0.91 | 1.70 |
| 0.92 | 1.75 |
| 0.93 | 1.81 |
| 0.94 | 1.88 |
| 0.95 | 1.96 |
| 0.96 | 2.05 |
| 0.97 | 2.17 |
| 0.98 | 2.33 |
| 0.99 |  |

## Finding Confidence Intervals E for a Proportion

$$
\begin{aligned}
& E=z_{\alpha / 2} \sqrt{\frac{\hat{p} \hat{q}}{n}} \text { proportion }(\hat{p} \text { and } \hat{q} \text { are known) } \\
& E=z_{\alpha / 2} \sqrt{\frac{\hat{p} \hat{q}}{n}} \text { proportion (use } 0.25 \text { when } \mathrm{p} \text { and } \mathrm{q} \text { are not known) }
\end{aligned}
$$

Sample: A research poll 200 respondents were asked if they felt vulnerable to identity theft. Thirty said yes. Find the confidence interval given a confidence level of $95 \%$.
$\mathrm{N}=200 \mathrm{x}=30$ confidence interval of $95 \% \quad z_{\alpha / 2}=1.96 \quad \mathrm{p}=\frac{30}{200}=.15 \quad \mathrm{q}=1-.15=.85$

$$
E=z_{\alpha / 2} \sqrt{\frac{\hat{p} \hat{q}}{n}}=1.96 \sqrt{\frac{.15 \times .85}{200}}=0.4949
$$

Using the calculator:

Press STAT and the right arrow twice to select TESTS.

Use the down arrow to select
A:1-PropZInt...
Press ENTER.

Enter number of favorable outcomes, x , sample size, n , and confidence level.

Use left arrow to select Calculate and press ENTER.

Results:
$95 \%$ confidence interval is 0.10051 to 0.19949
when $\mathrm{P}=0.15$ and sample size is 200 .

$$
E=\frac{.19949-.10051}{2}=\frac{.09898}{2}=0.4949
$$

EDIT CALC TESTS
A: 1-PropZInt

$$
\begin{aligned}
& \text { 1-PropZInt } \\
& \text { x: } 30 \\
& \text { n: } 200 \\
& \text { C-Level:.95 } \\
& \text { Ealculate }
\end{aligned}
$$

$$
\begin{aligned}
& 1 \text { - PropZInt } \\
& \text { (.10051,19949) } \\
& \hat{p}=.15 \\
& n=200
\end{aligned}
$$

## To find a confidence interval for Mean (known $\sigma$ )

$E=z_{\alpha / 2} \frac{\sigma}{\sqrt{n}}$
Example: A sample of 5 students were asked if they felt their test was fair. Three said yes. Find the confidence interval given a confidence level of $95 \%$ and a $\sigma$ of 5 .
$\sigma=5$, mean $=3, n=5$, Confidence level $=.95$
$z_{\alpha / 2}=1.96$
$E=z_{\alpha / 2} \frac{\sigma}{\sqrt{n}}=1.96 \frac{5}{\sqrt{5}}=4.3827=4.383$
Or use the calculator:

Press STAT and the right arrow twice to select TESTS.

Use the down arrow to select 7:Zinterval...
EDIT CALC TESTS
7: ZInterval

ZInterval
Enter population standard deviation, mean, sample size, and confidence level.
Press down arrow to select Calculate and press ENTER.

Results:
The 95\% confidence interval for Distribution is 1.38 to 7.83 when the mean is 3 the population standard deviation is 5.0 , and the sample size is 5 .
$E=\frac{7.3826-(-1.3837)}{2}=4.383$


$$
\begin{aligned}
& Z \text { Interval } \\
& \begin{array}{l}
(-1.3837, ~ 7.3826) \\
\bar{x}=3 \\
n=5
\end{array}
\end{aligned}
$$

## To find a confidence interval for Mean (unknown $\sigma$ )

$$
E=t_{\alpha / 2} \frac{s}{\sqrt{n}}
$$

Example: Find the confidence interval for a sample of 4 students given a sample mean of 6.8, a sample standard deviation of 1 and a confidence level of $95 \%$

$$
\begin{aligned}
& t_{\alpha / 2}=3.182 \quad \text { from T Chart for df } 3 \\
& E=t_{\alpha / 2} \frac{s}{\sqrt{n}}=3.182 \frac{1}{\sqrt{4}}=1.591
\end{aligned}
$$

Or use the calculator:
Press STAT and the right arrow twice to select TESTS.

Use the down arrow to select 8:TInterval...

EDIT CALC TESTS
8: TInterval


T Interval
(5.2088, 8.3912)
$x=6.8$
$\mathrm{Sx}=1.0$
$n=4.0$

## Determine Minimum sample size

## Proportion ( $\hat{p}$ and $\hat{q}$ are known)

Sample question: $41 \%$ of Jacksonville residents said that they had been in a hurricane. How many adults should be surveyed to estimate the true proportion of adults who have been in a hurricane, with a $95 \%$ confidence level and an interval of $6 \%$ ?
$n=\frac{\left[z_{\alpha / 2}\right]^{2} \hat{p} \hat{q}}{E^{2}} \quad$ proportion $(\hat{p}$ and $\hat{q}$ are known)
$n=\frac{[1.96]^{2}(.41)(.59)^{\wedge}}{.03^{2}}=1032.536711=1032.536711=1033$

## Proportion ( $\hat{p}$ and $q$ are not known)

Sample question: Jacksonville residents said that they had been in a hurricane. How many adults should be surveyed to estimate the true proportion of adults who have been in a hurricane, with a $95 \%$ confidence level and an interval of $6 \%$ ?
$n=\frac{\left[z_{\alpha / 2}\right]^{2} \bullet 0.25}{E^{2}}$ proportion $(\hat{p}$ and $\hat{q}$ are not known $)$
$n=\frac{[1.96]^{2}(.25)^{\wedge}}{.03^{2}}=1067.1111=1067.1111=1068$
Note: When finding the minimum sample size, always round up.

## Proportion (known standard deviation of the population $\sigma$ )

Sample question: Suppose we want to know the average age of a Florida State College student, plus or minus 0.5 years. We'd like to be $99 \%$ confident about our result. From a previous study, we know that the standard deviation for the population $\sigma$ is 2.9.

$$
\begin{aligned}
& n=\left[\frac{z_{\alpha / 2} \sigma}{E}\right]^{2} \\
& n=\left[\frac{2.575(2.9)}{.5}\right]^{2}=223.054225=224
\end{aligned}
$$

Note: When finding the minimum sample size, always round up.

## Chapter 8:

## Statistical Hypotheses

The best way to determine whether a statistical hypothesis is true would be to examine the entire population. Since that is often impractical, researchers typically examine a random sample from the population. If sample data are not consistent with the statistical hypothesis, the hypothesis is rejected.

There are two types of statistical hypotheses.

- Null hypothesis. The null hypothesis, denoted by $\boldsymbol{H}_{0}$, is usually the hypothesis that sample observations result purely from chance.
$\checkmark \quad H_{0}$ is always $=$.
- Alternative hypothesis. The alternative hypothesis, denoted by $\boldsymbol{H}_{\mathbf{1}}$, is the hypothesis that sample observations are influenced by some non-random cause.
$\checkmark \quad \boldsymbol{H}_{1}$, is never $=$. Options are $<,>, \neq$.
Chart for establishing $\boldsymbol{H}_{0}$ : $P$ and $H_{1}: P$

| Claim | $\mathrm{H}_{0}: \mathrm{P}$ | $\mathrm{H}_{1}: \mathrm{P}$ | Comments |
| :--- | :--- | :--- | :--- |
| $\mathrm{P}>5$ | $\mathrm{H}_{0}: \mathrm{P}=5$ | $\mathrm{H}_{1}: \mathrm{P}>5$ |  |
| $\mathrm{P}<5$ | $\mathrm{H}_{0}: \mathrm{P}=5$ | $\mathrm{H}_{1}: \mathrm{P}<5$ |  |
| $\mathrm{P}=5$ | $\mathrm{H}_{0}: \mathrm{P}=5$ | $\mathrm{H}_{1}: \mathrm{P} \neq 5$ | If difference between the assumption and the claim is close. <br> For instance, Claim $20 \%$ of adults smoke. Survey showed $21 \%$ <br> of respondents smoke. |
| $\mathrm{P} \neq 5$ | $\mathrm{H}_{0}: \mathrm{P}=5$ | $\mathrm{H}_{1}: \mathrm{P} \neq 5$ | If difference between the assumption and the claim is close. <br> For instance, Claim $20 \%$ of adults smoke. Survey showed $21 \%$ <br> of respondents smoke. |
| $\mathrm{P} \geq 5$ | $\mathrm{H}_{0}: \mathrm{P}=5$ | $\mathrm{H}_{1}: \mathrm{P}<5$ | $\mathrm{H}_{1}$ is opposite the claim |
| $\mathrm{P} \leq 5$ | $\mathrm{H}_{0}: \mathrm{P}=5$ | $\mathrm{H}_{1}: \mathrm{P}>5$ | $\mathrm{H}_{1}$ is opposite the claim |

## Types of Hypothesis Tests - P Value One tail vs two tail

$H_{1}: P$ Left - Tailed test = Actual P value
$H_{1}: P>$ Right - Tailed test $=1$-actual $P$ value
$H_{1}: P \neq$ Two tailed test. If test is a 2 tail, you have to double your calculated $P$ value

## Acceptance/Rejection

| Condition | Conclusion |
| :--- | :--- |
| Original claim does not include equality and you <br> reject $H_{0}$ because the $p$ value is <or $=$ to given <br> Critical value $\alpha$. | There is sufficient evidence to support the original <br> claim |
| Original claim does not include equality and you <br> fail to reject $H_{0}$ because the $p$ value is > than the <br> given Critical value $\alpha$. | There is not sufficient evidence to support the <br> original claim |
| Original claim includes equality and you reject $\mathrm{H}_{0}$ <br> because the $p$ value is < or $=$ to the given Critical <br> value $\alpha$. | There is sufficient evidence to warrant rejection of <br> the original claim |
| Original claim includes equality and you fail to <br> reject Ho because the $p$ value is $>$ than the given <br> Critical value $\alpha$. | There is not sufficient evidence to warrant <br> rejection of the original claim |

SUPPORT
ENTER

## Hypothesis Tests for Proportions (Test Statistic, P value and sample proportion).

$$
z=\frac{p-p}{\sqrt{\frac{p q}{n}}}
$$

Example: In sampling 200 people, we found that $30 \%$ of them favored a certain candy. Use $\alpha=$ $10 \%$ to test the hypothesis that the proportion of people who favored that candy is less than $35 \%$.

Solution: This represents a one-sample test of proportion. So we use the "1-PropZTest" function. The sample proportion is $30 \%$ or $p=0.30$, and the hypothesized claim is $p \geq 0.35$. Therefore, the hypotheses are $\mathrm{H}_{0}: p=0.35$ and $\mathrm{H}_{1}: p<0.35$. This is a two tail test because the clam includes an equal sign.
[NOTE: If the $p$-value $<\alpha$, reject the null hypothesis; otherwise, do not reject the null hypothesis].

$$
z=\frac{p-p}{\sqrt{\frac{p q}{n}}}=\frac{.30-.35}{\sqrt{(.35)(.65) / 200}}=-1.48 \quad \mathrm{P}=.0694
$$

This is a left tail test because $\boldsymbol{H}_{1}: \boldsymbol{P}$ < so $P=.0694$

## Use the Calculator:

Press STAT and the right arrow to select TESTS.
Use the down arrow to select 5.1-PropZTest

## Press ENTER

Enter: $\mathrm{H}_{0} p=0.35$, number of favorable outcomes $(\mathrm{x})$, sample size ( n ), and select the alternate hypothesis $p<0.35$

Use down arrow to select Calculate and press

## ENTER

Results:
The Test Statistic is -1.48
Since the $p$ value $=0.069$ is less than $\alpha=0.10$, reject the null hypothesis $\mathrm{H}_{0} p=0.35$

The hypothesized claim is $p \geq 0.35$ includes

## EDIT CALC TESTS

## 5: 1-PropZTest

 equality. We rejected $\mathrm{H}_{0}$. and conclude that there is sufficient evidence to warrant rejection of the original claim.

## Hypothesis Tests for Proportions for a Mean（known $\sigma$ ）

$$
z=\frac{\bar{x}-u}{\sigma / \sqrt{n}}
$$

Example：A sample of size 200 has a mean of 20 and a standard deviation of 16 and $\sigma=6$ ．Test the hypothesis claim that the population mean is not equal to19．2（ $\alpha=10 \%$ level of significance）．

$$
z=\frac{\bar{x}-u}{\sigma / \sigma}=\frac{19.2-20}{6 /}=-1.8856 \quad \mathrm{P}=.02968 \text { from chart }
$$

This is a two tail test $\left(\mathrm{H}_{1}: \mathrm{P} \neq\right)$ so $\mathrm{P}=2 \times .02968=.05936$

## OR Use the Calculator：

Solution：＂The population mean is $\neq 19.2$ means the null hypothesis is $\mathrm{H}_{0}=19.2$ ，and the alternate hypothesis is $\mathrm{H}_{1} \neq 19.2$ ．
［NOTE：If the $p$－value $<\alpha$ ，reject the null hypothesis；otherwise，do not reject the null hypothesis］．
Press STAT and the right arrow twice to select TESTS．

Use the down arrow to select 1：Z－Test
EDIT CALC TESTS
1：Z－Test

## Press ENTER．

Use right arrow to select Stats
Enter given hypothesized mean，standard deviation，mean，and sample size．
Select alternate hypothesis．
Press down arrow to select Calculate and press ENTER．

Results：
Since the p－value is 0.059346 is less than $\alpha=$ 0.10 ，reject the null hypothesis $\mathrm{H}_{0}=19.2$ ．

$$
\begin{aligned}
& \text { Z-Test } \\
& \text { InFtData St.ats } \\
& \text { 从口:19.2 } \\
& \text { 军: } \\
& \text { n: 200 }
\end{aligned}
$$

$$
\begin{aligned}
& \text { Ealculate Or aw }
\end{aligned}
$$

$$
\begin{aligned}
& Z \text { Test } \\
& u \neq 19.2 \\
& Z=1.885618083 \\
& P=.059346 \\
& \bar{x}=20.0 \\
& n=200
\end{aligned}
$$

## Using the T-Chart

Example: A researcher claims that the mean speed of a Miss Williams tennis serve is 125 mph . Therefore, $H_{0}=125$ and the alternate hypothesis is $H_{1} \neq 125$. Assume that data is normally distrusted for a sample test of 40 serves and the value of $\alpha$ is unknown. Find the critical values for the hypothesis test with a significance level of 0.05 . This is a two tail test because equal is included in the claim.

Go to the Table 3 Distribution Critical t values. Look in the row for a degree of freedom (df) of 39 (40-1) and the column for two tail significance level of 0.05 . The answer is $\pm 2.023$.

## This is a two tail.

Under two tail the values are:

| Two Tail | 0.01 | 0.02 | 0.05 | 0.10 | 0.20 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| df 39 | 2.708 | 2.426 | $\mathbf{2 . 0 2 3}$ | 1.685 | 1.304 |

## In the row for degree of freedom of 39, the number on the line for two tail of 0.05 .

Example: A researcher claims that the mean speed of a Miss Williams tennis serve is $>125 \mathrm{mph}$. Therefore, $H_{0}=125$ and the alternate hypothesis is $H_{1}>125$. Assume that data is normally distrusted for a sample test of 40 serves and the value of $\alpha$ is unknown. Find the critical values for the hypothesis test with a significance level of 0.05 . This is a one tail test because equal is not included in the claim.

Go to the Table 3 Distribution Critical $t$ values. Look in the row for df is 39 (40-1) and the column for one tail significance level of 0.05 . The answer is 1.685 .

Under one tail the values are:

| OneTail | 0.005 | 0.01 | 0.025 | 0.05 | 0.10 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| df 39 | 2.708 | 2.426 | 2.023 | 1.685 | 1.304 |

## The P-value for a right-tailed test when the given test statistic is not on the chart.

Example with $\mathrm{n}=28$ and test statistic $\mathrm{t}=3.026$.
The degree of freedom is $28-1=27$. This is a one tail test.
On the t-Distribution chart go to the row for degree of freedom of 27, the highest number on this line, 2.771 corresponds to a one tail of 0.005 .

| One Tail | 0.005 | 0.01 | 0.025 | 0.05 | 0.10 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| df 27 | 2.771 | 2.473 | 2.052 | 1.703 | 1.314 |

The $t=3.026$ given in the problem is higher than the highest value of 2.771 for .005 , therefore the probability must be less than 0.005 .

Go to $2^{\text {nd }}-$ VARS $-T c d f$ hit ENTER
Tcdf (lower, highest, df)

After Tcdf enter (3.026, 9999, 27)
Hit ENTER P value = 0026951941

## Hypothesis Tests for a Mean (unknown $\sigma$ )

## Find test statistic and $p$ value of a T-Test

Example: A sample of size 20 has a mean of 110 and a standard deviation of 16. Use the TI-83 calculator to test the hypothesis claim that the population mean is greater than 100 with a level of significance of $\alpha=5 \%$.

$$
t=\frac{\bar{x}-u}{s / \sqrt{n}}=\frac{100-110}{16 / \sqrt{20}}=-2.7951
$$

Use technology to find the P -value for a one-tailed test with $\mathrm{n}=20$ and test statistic $\mathrm{t}=-2.7951$ This is a T test. On the $t$ chart for df of 24 the value 2.797 does not exist.

## Go to $2^{\text {nd }}-$ VARS - Tcdf hit ENTER

> Tcdf (lower, highest, df)

> Tcdf (-9999, -2.7951, 19)

Enter you data , scroll down to Paste and press ENTER
Press $\boldsymbol{E N T E R}$ again to see your answer $\mathrm{P}=.0057731153$

Solution: "The population mean is greater than 100 means the alternate hypothesis is $\mathrm{H}_{1}: \mu>100$, and the null hypothesis is $\mathrm{H}_{0}: \mu=100$.
Press STAT and the right arrow twice to select TESTS.

To select the highlighted
2: T-Test...
Press ENTER.

EDIT CALC TESTS
2: T-Test

Use right arrow to select Stats (summary values rather than raw data) and Use the down arrow to Enter the hypothesized mean, sample mean, standard deviation, and sample size.
Select alternate hypothesis.
Press down arrow to select Calculate and press ENTER.

Result in both cases:
The test statistic $\mathrm{t}=2.795$.
Since the $p$-value is 0.0058 , is less than $\alpha=0.05$, reject the null hypothesis $H_{0} \mu=100$.

Original hypothesis claim that the population mean is greater than 100
 does not include equality and you reject $H_{0}$ because the $p$ value is $<$ or $=$ to given Critical value $\alpha$. There is sufficient evidence to support the original claim.

## Chapter 10:

## To find the linear correlation coefficient, $r$, $p$-value

Example: Given sample of size 5 and a level of significance of $\alpha=5 \%$. find the linear correlation, $r$ and $p$-value with a hypothesis of $>0$.

| x | 230 | 265 | 358 | 480 | 530 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| y | 15.9 | 15.7 | 15.4 | 15.3 | 14.9 |

Make sure the data for the $x$ data is entered in L1 and $y$ data is entered in L2.

Press STAT, TESTS, and press the down arrow to F:LinRegTTest. Press ENTER.

Select the alternate hypothesis $\mathbf{>} \mathbf{0}$.
Use the down arrow to get to calculate and press ENTER.

EDIT CALC TESTS
F: LinRegTTest

LinRe9TTest.
Frea: 1
B 人 $9: \%$ 人 0
RegE: $\mathrm{Y}_{1} \%$
Calculat.e

$$
\begin{aligned}
& \text { LinRegTTest } \\
& y=a+b x \\
& \beta>0 \text { and } p>0 \\
& t=-5.858137033 \\
& p=.9950411445 \\
& a=16.49091023 \\
& b=-.0028204784 \\
& s=.1259514044 \\
& r^{2}=.9196093432 \\
& r=-.9589626391
\end{aligned}
$$

Conclude that there is a positive relationship between the x and y values.

## Find the effects of an outlier

Review the scatterplot below:
a. Examine the pattern of all points and subjectively determine whether there appears to be a correlation between $x$ and $y$.


Given sample of size 10 and a level of significance of $\alpha=5 \%$.
The ten points on the scatterplot are shown in the table below:

| $x$ | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 3 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $y$ | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 10 |

b. Enter Data

STAT 1:Edit Clear Old Data [Omit this step if there is no old data in L1 and/or L2. Enter $«$ STAT 1:Edit (Enter input ( x ) values in L1 and output (y) values in L2.) Make sure every entry is correct.
c. Using technology find the value of the correlation coefficient $r$ and determine whether there is a linear correlation.

Press STAT, TESTS, and press the down arrow to F:LinRegTTest. Press ENTER.

## The Freq List should be 1 and StoreRegEq should be blank

## Scroll down to Calculate and hit ENTER

## EDIT CALC TESTS F: LinRegTTest

## LinReg

x List: L1
y List: L2
Frequency:List:1
Store RegEQ:
Calculate

Table 5 Linear Correlation for . 05 confidence level and $n=10$ is $\pm 0.632$.

So $r=.91$ falls in the critical area to the right outside of $\pm 0.632$ and there does appears to be linear correlation.

LinReg
$y=a+b x$
$a=.2641509434$
$b=.9056603774$
$s=1.19551047$
$r^{2}=.8202207191$
$r=.9056603774$

## Find the effects of an outlier (continued)

d Now remove the point $(10,10)$ from L1 and L2 and repeat parts a and b

| $x$ | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 3 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $y$ | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 |

e Using technology find the value of the correlation coefficient r and determine whether there is a linear correlation.

Choose STAT, TESTS, F:LinRegTTest
EDIT CALC TESTS
F: LinRegTTest

$$
\begin{aligned}
& \quad \text { LinReg } \\
& \boldsymbol{y}=\boldsymbol{a}+\boldsymbol{b x} \\
& a=2 \\
& b=0 \\
& s=.9258200998 \\
& r^{2}=0 \\
& r=0
\end{aligned}
$$

## Finding the equation of the Regression Line

Example: The data below shows the chest size and weight of 5 bears. Find the regression equation, letting chest size be the variable $x$ and weight be the variable $y$.

| x chest size | 40 | 53 | 38 | 43 | 44 | 58 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $y$ weight | 227 | 360 | 153 | 206 | 234 | 414 |

1. Clear Old Data [Omit Step 1 if there is no old data in L1 and/or L2.]
2. Enter Data for above chart

STAT 1:Edit (Enter input (x) values in L1 and output (y) values in L2.)
Make sure every entry is correct.
3 Choose STAT, CALC, 4:LinReg(ax + b).
EDIT CALC TESTS
4: LinRegT(ax+b)

## 4 Leave Freq List And Store Req blank. Use the down arrow To move to Calculate. Press ENTER

LinReg
$y=a x+b$
x List: L1
y List: L2
Frequency:List
Store RegEQ
Calculate

5 You will see the estimated slope (the value for a) and the estimated y intercept (the value for $b$ ).

> LinReg
> $y=a x+b$
> $a=12.47058824$
> $b=-307.9803922$

So the regression equation $y=a x+b$ is $y=12.47 x$-308
What is the best predicted weight of a bear with a chest size of 41 inches?

$$
y=12.47(41)-308=240.5027=203 \mathrm{lbs} .
$$

