Solving a System of Three Equations

with THREE VARIABLES each

Example:
$$\begin{aligned} x + y + z &= 2 \\ x - y + 2z &= 5 \\ 3x + 2y - z &= 3 \end{aligned}$$

1) Choose two of the 3 equations and add them to eliminate <u>one</u> of the variables…

$$\begin{aligned} x + \cancel{y} + z &= 2 \\ x - \cancel{y} + 2z &= 5 \\ \hline 2x \quad + 3z &= 7 \end{aligned}$$

Box the equation $\boxed{2x + 3z = 7}$

2) Next, choose two other equations (a different combination), and eliminate the <u>same</u> variable as you did in Step 1.

$$\begin{aligned} x - y + 2z &= 5 \\ 3x + 2y - z &= 3 \end{aligned}$$

Multiply this by 2

$$\begin{aligned} 2x - \cancel{2y} + 4z &= 10 \\ 3x + \cancel{2y} - z &= 3 \\ \hline \boxed{5x \quad + 3z = 13} \end{aligned}$$

Box this equation also

3) Take the two <u>boxed</u> equations and use them to eliminate another variable.

$$\begin{aligned} 2x + 3z &= 7 \\ 5x + 3z &= 13 \end{aligned}$$

Multiply this by -1

$$\begin{aligned} 2x + \cancel{3z} &= 7 \\ -5x - \cancel{3z} &= -13 \\ \hline -3x \quad &= -6 \end{aligned}$$

$x = 2$

4) Now that you found x, plug it back into one of the boxed equations to find another variable…

$$\begin{aligned} 5x + 3z &= 13 \\ 5(2) + 3z &= 13 \\ 3z &= 3 \\ z &= 1 \end{aligned}$$

5) Plug the x and z into one of the <u>original</u> equations to find the last variable.

$$\begin{aligned} x + y + z &= 2 \\ 2 + y + 1 &= 2 \\ y + 3 &= 2 \\ y &= -1 \end{aligned}$$

We find that x = 2 , y = -1, and z = 1.

Solution: (2, -1, 1)

Solving a System of Three Equations

with a MISSING VARIABLE

Example:

$$\begin{aligned} a + b + c &= 6 \\ a - b + 2c &= 5 \\ -a \quad\;\; - c &= -4 \end{aligned}$$

1. Choose the two equations that contain the variable that is missing from the third equation, and eliminate that variable from those two equations.

$$\begin{aligned} a + \cancel{b} + c &= 6 \\ \underline{a - \cancel{b} + 2c} &\underline{= 5} \\ 2a \;+\; 3c &= 11 \end{aligned}$$

2. Take the result from step 1 and pair it with the original equation that was missing a variable. Eliminate another variable…

$$\begin{aligned} 2a + 3c &= 11 \\ \underline{-a \;-\; c} &\underline{= -4} \end{aligned} \xrightarrow{\text{Multiply by 3}} \begin{aligned} -3a - \cancel{3c} &= -12 \\ \underline{2a + \cancel{3c}} &\underline{= 11} \end{aligned}$$

$$\xrightarrow[\text{to get a}]{\text{Multiply } -a \text{ by } -1} \begin{aligned} -a &= -1 \\ a &= 1 \end{aligned}$$

3. Now that you found the value of a, plug it into one of the two-variable equations and solve for another variable…

$$\begin{aligned} -a - c &= -4 \\ -1 - c &= -4 \end{aligned}$$

$$\xrightarrow[\text{to get c}]{\text{Multiply } -c \text{ by } -1} \begin{aligned} -c &= -3 \\ c &= 3 \end{aligned}$$

4. Plug the a and c into one of the original equations to find the last variable.

   We find that a = 1, b = 2, and c = 3.

$$\begin{aligned} a + b + c &= 6 \\ 1 + b + 3 &= 6 \\ b + 4 &= 6 \\ b &= 2 \end{aligned}$$

Solution: (1, 2, 3)