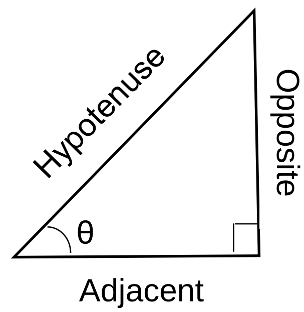
Right Triangle Trigonometric Functions

Definitions of the Trigonometric Functions

In a right triangle, trigonometric functions can be defined as ratios formed by the lengths of any two sides of the triangle. The hypotenuse is always the longest side of the right triangle, and the other two sides are called opposite or adjacent, based on where they are in relation to the angle being considered. The Greek letter theta (θ) is often used to identify the angle in the trigonometric ratios. Names and definitions of the ratios are listed below for the angle θ in the diagram at the right.



Sine θ = $\frac{\text{opposite}}{\text{hypotenuse}}$ Cosine θ = $\frac{\text{adjacent}}{\text{hypotenuse}}$ Tangent θ = $\frac{\text{opposite}}{\text{adjacent}}$

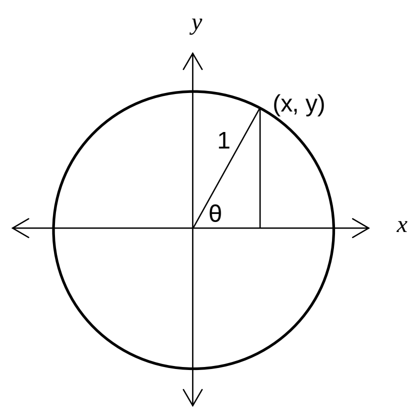
The Reciprocal Functions of Sine, Cosine and Tangent

Sine, cosine, and tangent, usually abbreviated as sin, cos and tan, are three of the most commonly used trig functions. The other functions - cosecant, secant and cotangent - are reciprocals of the three functions above and are abbreviated as csc, sec and cot.

$\csc\theta = \frac{1}{\sin\theta} = \frac{\text{hypotenuse}}{\text{opposite}}$ $\sec\theta = \frac{1}{\cos\theta} = \frac{\text{hypotenuse}}{\text{adjacent}}$ $\cot\theta = \frac{1}{\tan\theta} = \frac{\text{adjacent}}{\text{opposite}}$

Trigonometric Functions and the Unit Circle

The unit circle is a circle whose center is at the origin of the x-y coordinate plane, and has a radius of 1. When an acute angle θ is formed by the x-axis and a radius of the unit circle, it forms a right triangle with a hypotenuse of 1. Because the opposite side of angle θ has a length of y and the adjacent side of angle θ has a length of x, the functions sin, cos, and tan of θ can be determined easily, using the x and y coordinates of the point where the radius meets the unit circle.



$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{y}{1} = y$ $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{x}{1} = x$ $\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{y}{x}$

When the lengths of two sides of any right triangle are known, the Pythagorean theorem will help us find the length of the third side. If the hypotenuse of the triangle is a radius of the unit circle, the other two sides have lengths of x and y. That means $x^2 + y^2 = 1^2$

We saw above that the sine of angle θ is equal to y, and the cosine of θ is equal to x, so we can use $x^2 + y^2 = 1$ to derive a basic trigonometric identity: $\sin^2\theta + \cos^2\theta = 1$.