PROBABILITY

To calculate a probability when outcomes are equally likely, we can use the formula:

$$P(E) = \frac{n(E)}{n(S)}$$

This says the probability of an event E is equal to the number of outcomes of event E divided by the total number of outcomes in the sample space S.

Example: What is the probability that a total of 4 shows when we roll two fair dice?

The sample space for rolling two dice has 36 ordered pairs of numbers. We will represent the event "rolling a total of four" by F. Then the set of three possible outcomes of F = {(1,3), (2,2), (3,1)}.
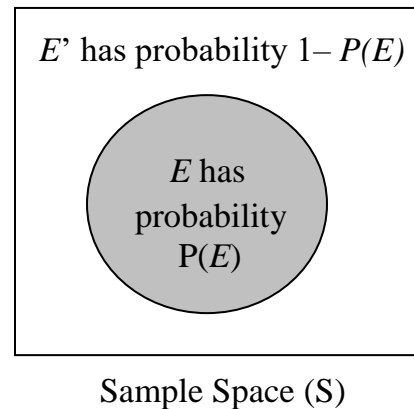
$$\text{Therefore,} \quad P(F) = \frac{n(F)}{n(S)} = \frac{3}{36} = \frac{1}{12}$$

To calculate the probability of the **complement** (labeled *E'*) of an event we can use the formula:

$$P(E') = 1 - P(E)$$

If the probability of *E* is .25, we can find the probability of the complement of *E* using:

$$P(E') = 1 - .25 = .75$$

E' has probability 1– *P(E)*

E has probability P(*E*)

Sample Space (S)

If you toss a fair coin, it will land on either heads or tails. Thus, the probability of heads, P(H), is $\frac{1}{2}$, as is the probability of tails. If we toss it twice, the outcome of the first toss does not influence the outcome of the second toss, and vice versa. For this reason, we call the two events **independent**.

What is the probability that it will come up heads on the first toss and heads on the second toss? Because the events are independent, we multiply the probability of the first toss by that of the second toss. So, the probability of heads occurring on each of two consecutive tosses is:

$$P(H \cap H) = P(H) \times P(H) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

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To find the probability of a **union** of two events (event E *or* event F), we can use the formula:

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

What is the probability that we draw either a heart **or** a face card from a standard 52 card deck?

$$P(H \cup F) = P(H) + P(F) - P(H \cap F) = \frac{13}{52} + \frac{12}{52} - \frac{3}{52} = \frac{22}{52} = \frac{11}{26}$$

We add the probability of a heart with the probability of a face card, and then subtract the probability of a heart that is also a face card to find our answer.

When we compute the probability of an event *F* assuming an event *E* has already occurred, this is called the **conditional probability** of *F*, given *E*. To find the conditional probability we can use the formula:

$$P(F \mid E) = \frac{P(E \cap F)}{P(E)} = \frac{n(E \cap F)}{n(E)}$$

We draw 1 face card from a 52 card deck, what is the probability that it is a queen?

$$P(Q \mid F) = \frac{P(F \cap Q)}{P(F)} = \frac{n(F \cap Q)}{n(F)} = \frac{4}{12} = \frac{1}{3}$$

To compute this, we take the probability of drawing a face card AND a queen, $P(F \cap Q)$, and divide it by the probability of drawing any face card *P(F)*.

All Face Cards

$F \cap Q$

Only Queens

JH,   KH,   QH,
JS,   KS,   QS,
JC,   KC,   QC,
JD   KD   QD
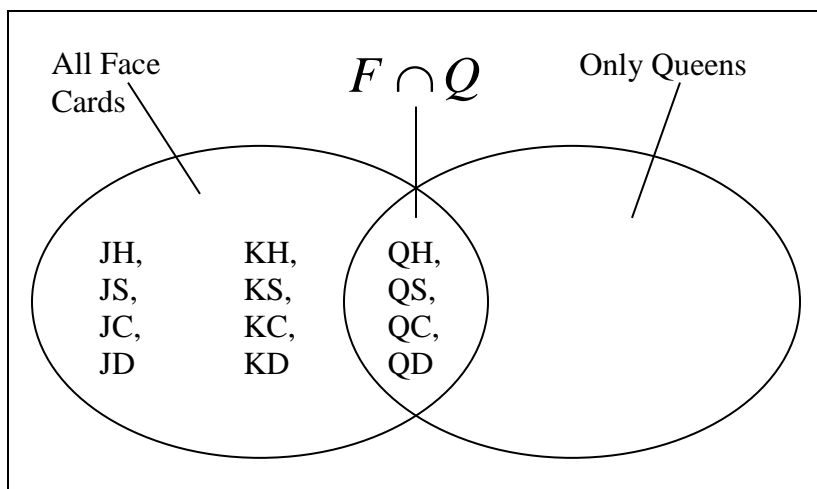
Guide to above diagram:

F = face card, J = Jack, K= King, Q = Queen

H = Heart, S = Spade, C = Club, D = Diamond