Solving a Quadratic Equation

By completing the square

There are some quadratic equations that are not factorable with integers, and you might think they cannot be solved… Take, for example: $x^2 - 8x = 21$

You cannot solve this equation by factoring, but you can either *Complete the Square*, or use the *Quadratic Formula* (page 2 of this handout). Completing the square is also used in College Algebra and Pre-Calculus.

Solving the equation: $x^2 - 8x = 21$

To solve this equation, first we put it in the form $ax^2 + bx + c = 0$, known as the *standard form* of a quadratic equation.

$$x^2 - 8x - 21 = 0$$

Divide the coefficient of the middle term (–8) by 2, and square the result.

$$-8/2 = -4$$

$$(-4)^2 = 16$$

Add this quantity (16) to each side of the equation.

$$x^2 - 8x - 21 + 16 = 0 + 16$$

Group the first two terms with the newly added quantity in a set of parentheses.

$$(x^2 - 8x + 16) - 21 = 16$$

Factor this trinomial grouping into its perfect square factors.

$$(x - 4)(x - 4) - 21 = 16$$

Add (or subtract, if necessary) the quantity outside the parentheses (21) on both sides of the equation.

$$(x - 4)(x - 4) = 16 + 21$$

Simplify the expression, writing the squared term with an exponent.

$$(x - 4)^2 = 37$$

Now we solve for x using the square root property.
Take the square root on both sides of the equation, and be sure to recognize that the result can take either sign.
(To show this, we use ± in front of the radical symbol.)

$$\sqrt{(x-4)^2} = \pm\sqrt{37}$$

$$(x - 4) = \pm\sqrt{37}$$

The x – 4 comes out from under the radical symbol. The 37 will stay because it is not a perfect square, and it cannot be simplified. The ± symbol indicates two solutions, so we solve for both.

$$x - 4 = +\sqrt{37} \qquad x - 4 = -\sqrt{37}$$

$$x = 4 + \sqrt{37} \qquad x = 4 - \sqrt{37}$$

Solving a Quadratic Equation

With the quadratic formula

Another way to solve un-factorable quadratic equations is to use the *quadratic formula*. Some students think that this method is foolproof, as it can be used with all quadratic equations. One of the desirable qualities of the quadratic formula is that, like many formulas, you simply plug numbers into it, and then follow the order of operations until you get the result.

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

The Quadratic Formula

We will use the example from above, expecting we should get the same answer when solving.

$$x^2 - 8x = 21$$

As we did earlier, we first want to put the equation in the form $ax^2 + bx + c = 0$, the standard form of a quadratic equation. Identify each of the values for "a", "b", and "c". In our example:

$$x^2 - 8x - 21 = 0$$

a = 1, b = –8 and c = –21.

Now we use the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(which you should memorize), and

plug in values of a(1), b(-8), and c(-21).

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4\,(1)(-21)}}{2(1)}$$

Follow the order of operations to solve for x:

$$x = \frac{-(-8) \pm \sqrt{64 - (-84)}}{2(1)}$$

Simplify the radical: Simplify the fraction:

$$x = \frac{8 \pm \sqrt{148}}{2} \longrightarrow x = \frac{8 \pm 2\sqrt{37}}{2} \longrightarrow x = \frac{8}{2} \pm \frac{2\sqrt{37}}{2} \longrightarrow x = 4 \pm \sqrt{37}$$

This means that $x = 4 + \sqrt{37}$ and $x = 4 - \sqrt{37}$ are the two solutions.

Sometimes, there will be only one solution, and sometimes there are no real number solutions.

The number of solutions depends on the value of the *discriminant.* The *discriminant* is the quantity under the radical symbol in the quadratic formula, $b^2 - 4ac$. When:

$b^2 - 4ac > 0$	there will be **2 real number** solutions
$b^2 - 4ac = 0$	there will be **1 real number** solution
$b^2 - 4ac < 0$	there will be **2 imaginary number** solutions (see Imaginary numbers handout for a refresher)