Imaginary Numbers

The equation $x^2 + 25 = 0$ has no solution in the set of real numbers. In order to solve such equations, it is necessary to go outside of the real number system by defining an Imaginary Number ***i**.*

The symbol ***i*** represents an imaginary number with a definition that says $i = \sqrt{-1}$. So, that means $i^2 = -1$.

Using this new number *i*

$$\sqrt{-25} = \sqrt{-1 \cdot 25} = \sqrt{-1} \cdot \sqrt{25} = \mathrm{i}\sqrt{25} = 5\mathrm{i}$$

$$\sqrt{-36} = \sqrt{-1 \cdot 36} = \sqrt{-1} \cdot \sqrt{36} = \mathrm{i}\sqrt{36} = 6\mathrm{i}$$

For any positive real number n, $\sqrt{-\mathrm{n}} = \mathrm{i}\sqrt{\mathrm{n}}$.
Imaginary numbers are not real numbers, and some properties of real numbers do not apply to imaginary numbers.

One such property is the Product Rule for Radicals, $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$ only applies if $\sqrt[n]{a}$ and $\sqrt[n]{b}$ exist among the set of real numbers.

The product rule does not apply to $\sqrt{-25} \cdot \sqrt{-4}$ because $\sqrt{-25}$ and $\sqrt{-4}$ do not define real numbers. $\sqrt{-25} \cdot \sqrt{-4} = \mathrm{i}\sqrt{25} \cdot \mathrm{i}\sqrt{4} = 5\mathrm{i} \cdot 2\mathrm{i} = 10\mathrm{i}^2 = -10$

Note: *In this case, if you were to apply the Product Rule of Radicals you would get 10 instead of -10.*

Powers of *i*

$i^1 = \sqrt{-1} = i$

$i^2 = \sqrt{-1} \times \sqrt{-1} = -1$

$i^3 = i^2 \times i^1 = -1 \times i = -i$

$i^4 = i^2 \times i^2 = (-1)(-1) = 1$

$i^5 = i^4 \times i^1 = 1 \times i = i$

$i^6 = i^4 \times i^2 = 1 \times (-1) = -1$

$i^7 = i^4 \times i^3 = 1 \times (-i) = -i$

$i^8 = i^4 \times i^4 = 1 \times 1 = 1$

As the exponent on *i* increases by 1, this pattern (*i*, -1, -*i*, 1) repeats for all positive integer powers of *i*.