Student's t-distribution

(Testing a Hypothesis about a Population Mean when σ is unknown)

Student's *t*-distribution (or simply the ***t*-distribution**) is a probability distribution that arises in the problem of estimating the mean of a normally distributed population when the population standard deviation is unknown and has to be estimated from the data.

Table A-3 T Distribution

			α			
Degrees of Freedom	.005 (one tail) .01 (two tail)	.01 (one tail) .02 (two tail)	.025 (one tail) .05 (two tail)	.05 (one tail) .10 (two tail)	.10 (one tail) .20 (two tail)	.25 (one tail) .50 (two tail)
1	63.657	31.821	12.706	6.314	3.078	1.000
2	9.925	6.965	4.303	2.920	1.886	.816
3	5.841	4.541	3.182	2.353	1.638	.765
4	4.604	3.747	2.776	2.132	1.533	.741
5	4.032	3.365	2.571	2.015	1.476	.727
6	3.707	3.143	2.447	1.943	1.440	.718
7	3.500	2.998	2.365	1.895	1.415	.711
8	3.355	2.896	2.306	1.860	1.397	.706
9	3.250	2.821	2.262	1.833	1.383	.703
7	3.500	2.998	2.365	1.895	1.415	.711
8	3.355	2.896	2.306	1.860	1.397	.706
9	3.250	2.821	2.262	1.833	1.383	.703
10	3.169	2.764	2.228	1.812	1.372	.700
11	3.106	2.718	2.201	1.796	1.363	.697
39	2.708	2.426	2.023	1.685	1.304	.681
40	2.704	2.423	2.021	1.684	1.303	.681
50	6.678	2.403	2.009	1.676	1.299	.679

Problem: According to the Centers for Disease Control, the mean number of cigarettes smoked per day by individuals who are daily smokers is 18.1. A researcher claims that retired adults smoke less than the general population of daily smokers. He obtains a random sample of 40 retired adults who are smokers and records the number of cigarettes smoked daily. The sample mean was 16.8 cigarettes and the standard deviation was 4.7 cigarettes. Test the claim at a $\alpha = 0.1$ level of significance that retired adults who smoke daily smoke less than the general population of daily smokers?

Step 1: The researcher wants to know if retired adults smoke less than the general population. The mean number of cigarettes smoked per day by individuals who are daily smokers is 18.1. This means

$$H_0 : \mu = 18.1$$
$$H_1 : \mu < 18.1$$

The key word for the alternative hypothesis is adults who smoke daily smoke **LESS** than the general population. This is a left-tailed test.

Step 2: The level of significance is α = 0.1. The degrees of freedom, df = n – 1 = 40 – 1 = 39

Look at the t-distribution where df=39 and α = 0.1 in one tail. The critical value is t = 1.304. Since the t-distribution is symmetric, the critical value is t = -1.304 for a left tail test.

Step 3: The sample mean is $\bar{X} = 16.8$ and the sample standard deviation is s = 4.7.

The test statistics is $$t = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{n}}} = \frac{16.8 - 18.1}{\frac{4.7}{\sqrt{40}}} = -1.749$$

Step 4: Since the test statistic -1.749 < the critical value -1.304

Conclusion: Reject H_0. The sample data supports the claim that retired adults who smoke daily smoke less than the general population of daily smokers.